Applications of Logarithms in Real Life

Kanagawa Prefectural Hadanosoya High School

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Hello, I’m going to talk about a math class I teach, the Applications of Logarithms in Real Life.

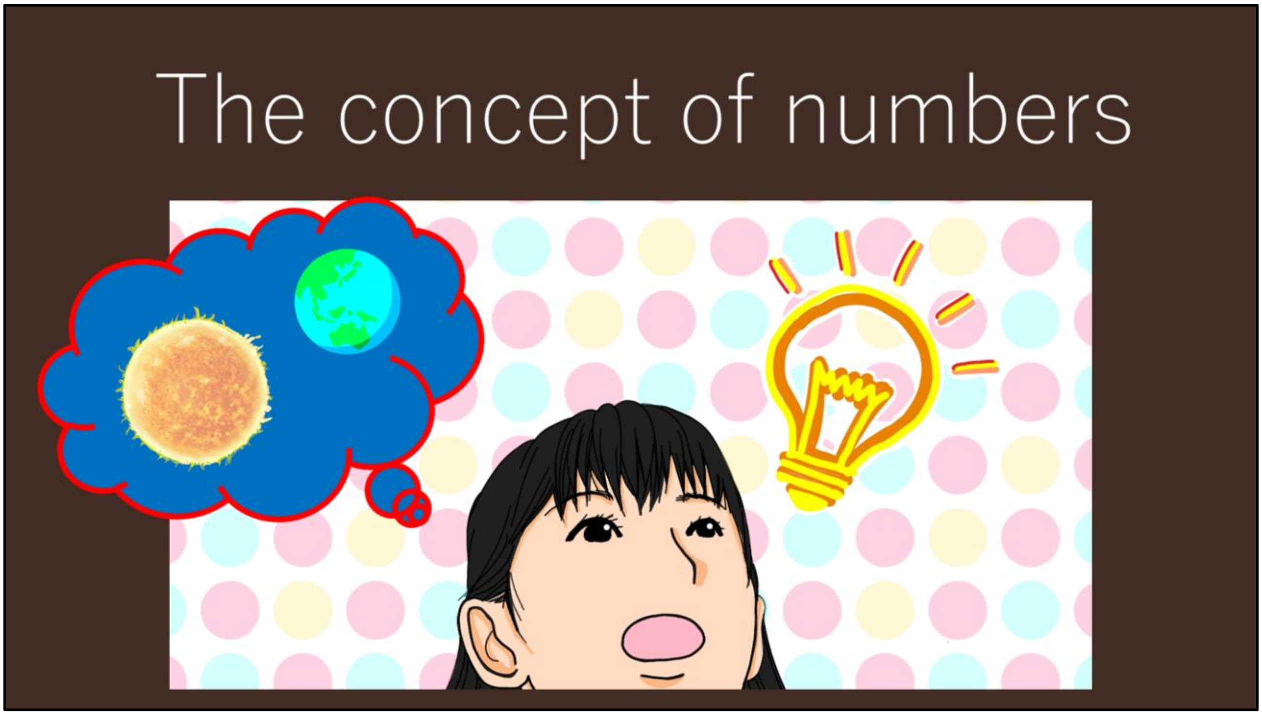
Self-introduction

- Name: Mami KIKUCHI
- Organization: Hadanosoya high school
- Subject: math

First, let me introduce myself.
I'm Mami KIKUCHI.
I have been working as a math teacher at Hadanosoya high school for 5 years.
Hadanosoya high school is a place full of nature.
If the weather is clear, we can see Mt Fuji from the roof balcony on the third floor.

Hadanosoya high school was designated to promote the improvement of teaching abilities and create the most effective methods for classroom interactions.

My lesson about the universe was carried out as a part of the projects to improve our pedagogy.
I hope, you will think, "Mathematics is fun!" and "The universe is so interesting!" at the end of my presentation.



The objective of today’s lesson is to make a quick and accurate comparison of size.

Can you imagine a large number that you can’t scale with a computing device?
Can you imagine a small number that you can’t see with your eyes?

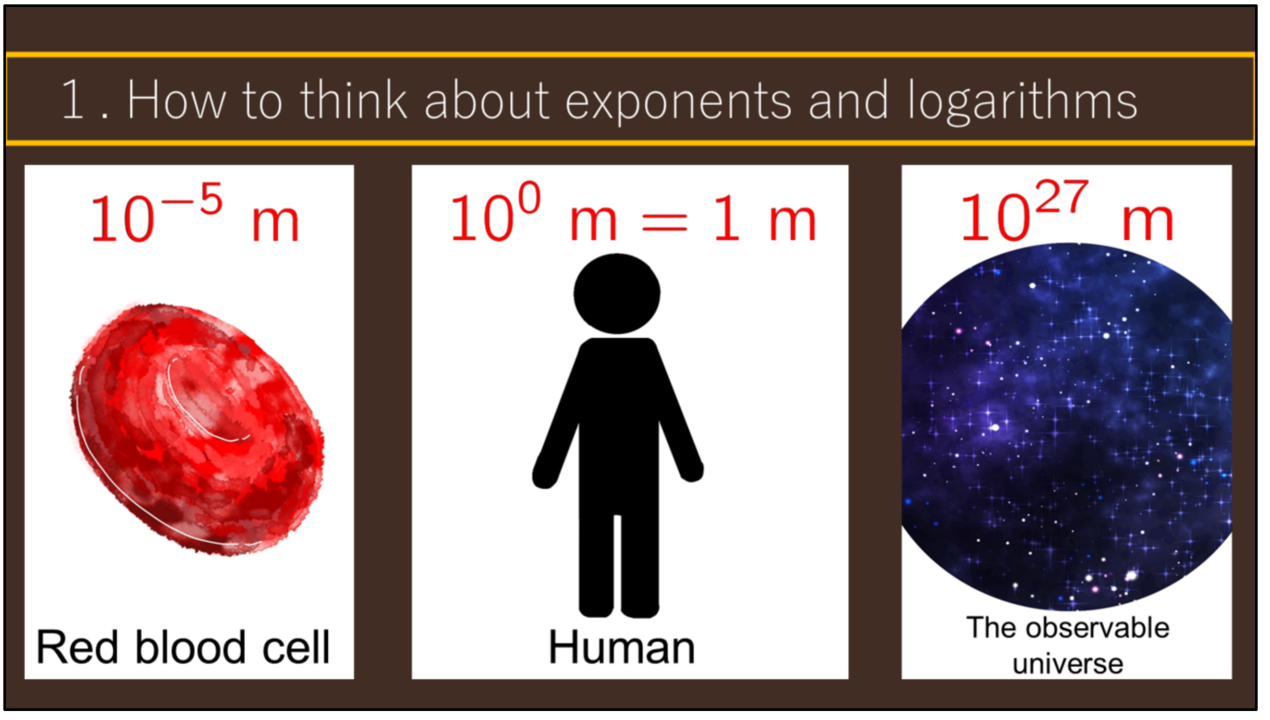
Today we are going to compare and understand the size of things.
It’s gonna be so much fun!
I hope you’ll find it interesting and useful.

Topics

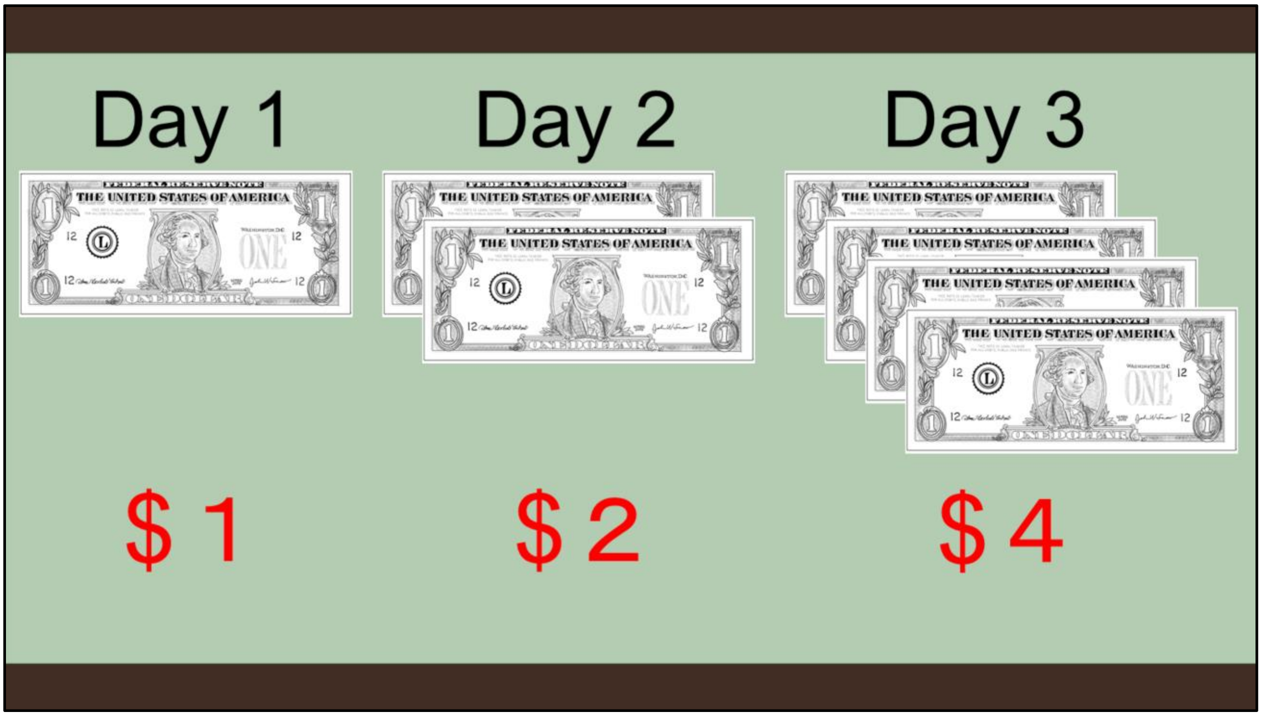
1. How to think about exponents and logarithms
2. More about the tool that makes us feel numbers real
3. Some examples with tools
4. The students’ reactions
5. Conclusion

Now, let me begin today’s presentation.
I enjoyed teaching logarithms because the students feel themselves being closer to the universe.
And here is what I will teach you today.

First, how to think about exponents and logarithms.
Second, more about the tool that makes us feel numbers real.
Third, some examples with tools.
Fourth, the students’ reactions.
Lastly, conclusion.



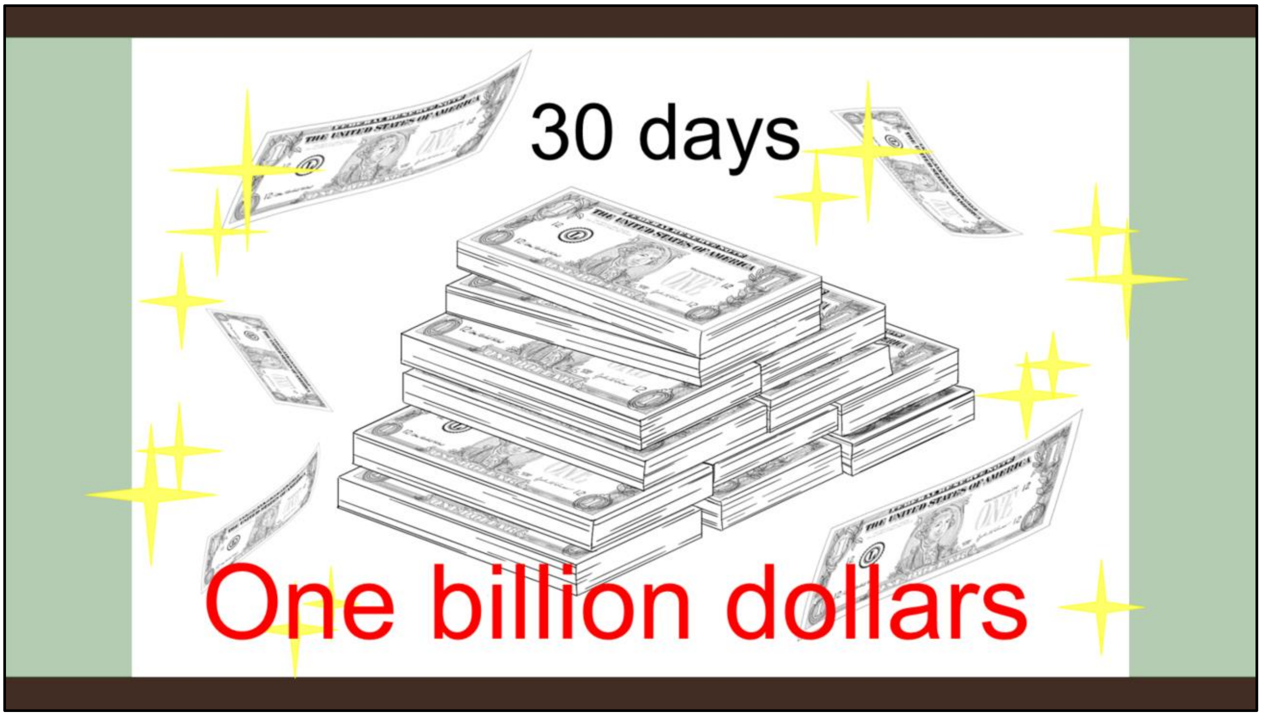
Large and small objects can be simply expressed using exponents.
For example, red blood cells are 10^{-5} m, a human being is 10^{0} m, and the observable universe is 10^{27} m.
Using exponents, objects can be easily compared by the number of digits.



Let me give you an example.
Let's imagine you get $1 for working the first day, $2 for working second days, and $4 for working third days at the Japan Aerospace Exploration Agency "JAXA".

When will you get $8?
（4days later!）
That's right!
The logarithm is what we use to find the answer.

You get $1 for working the first day.
If you work the next day, you get paid twice the day before.
How many days will it take for you to earn one billion dollars?

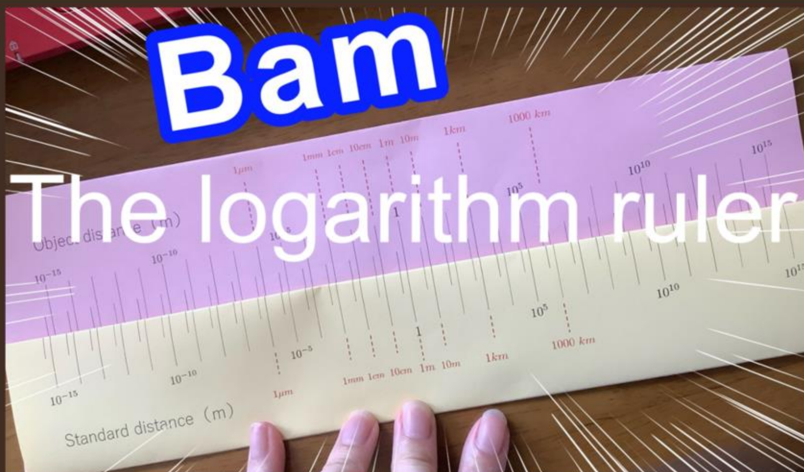


As it turns out, if you work 30 days, you will get a billion dollars.

JAXA is going bankrupt.

Logarithms can be used to find the number of times the multiplication is repeated.
In order to prevent JAXA from going bankrupt, it is necessary to grasp the concept of numbers.

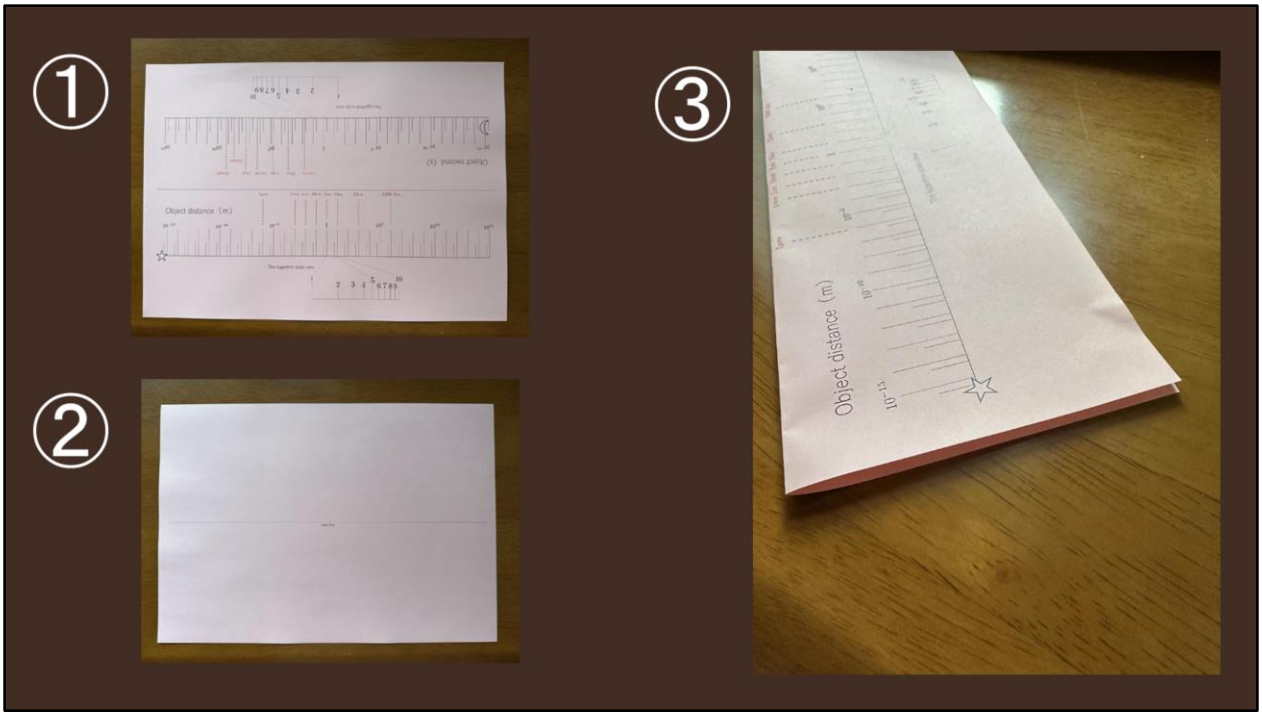
2. More about the tool that makes us feel it real



Now, let me show you a new and useful tool.

It is called the logarithm ruler, you can use it to identify how large or small a number is.

Now, let's make a logarithm ruler together.



First, take the pink paper.
To make the pink logarithm ruler, please fold the paper in half.

(Can you see two types of rulers printed on one side?
Fold the paper in half so the rulers are on the outside.)

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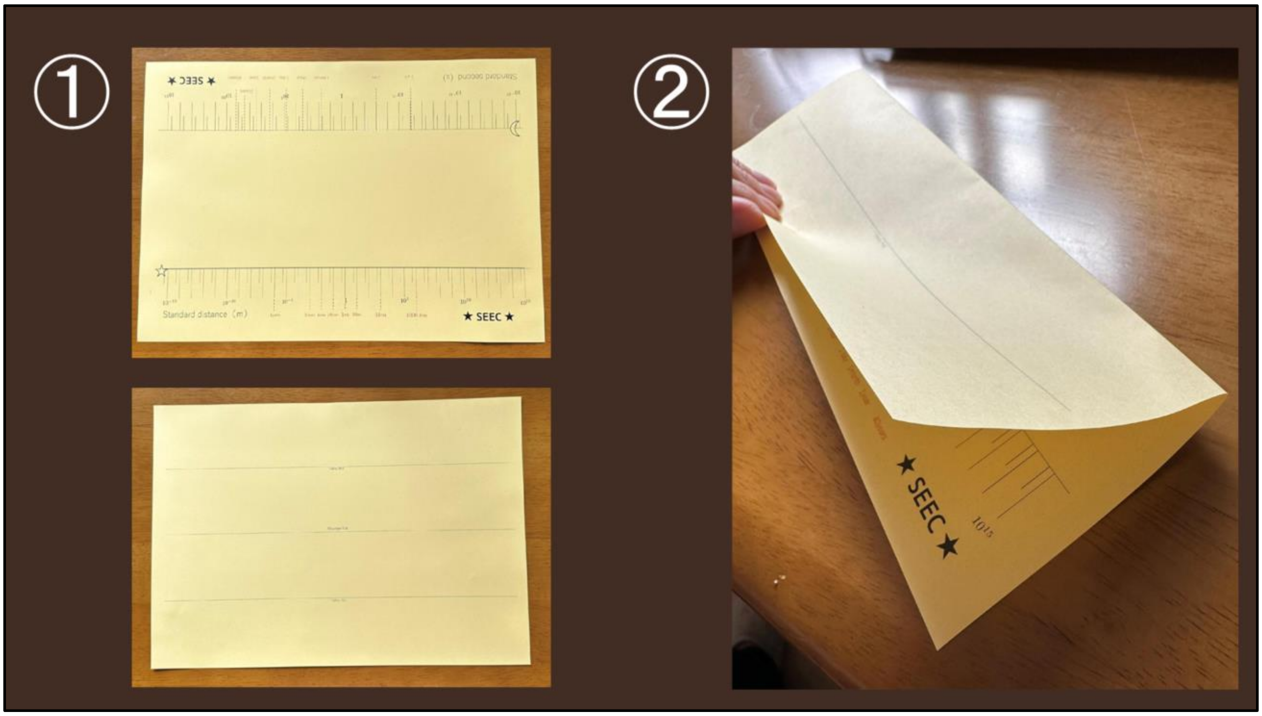

Next, Let’s make the yellow logarithm ruler.
In the same way, please fold it in half.

(Can you see two types of rulers printed on one side?
Turn it over, you can find the instruction of how to fold the paper.
Please, fold it in half. So that, the rulers are on the inside.)

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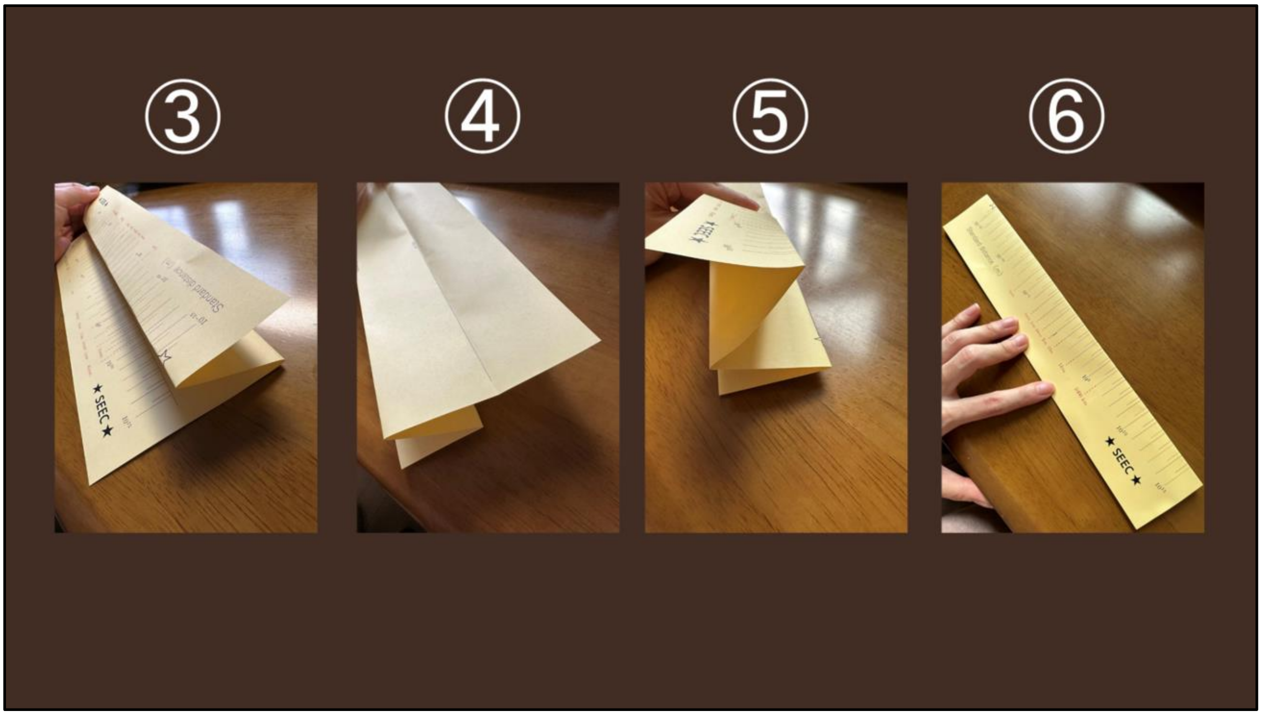
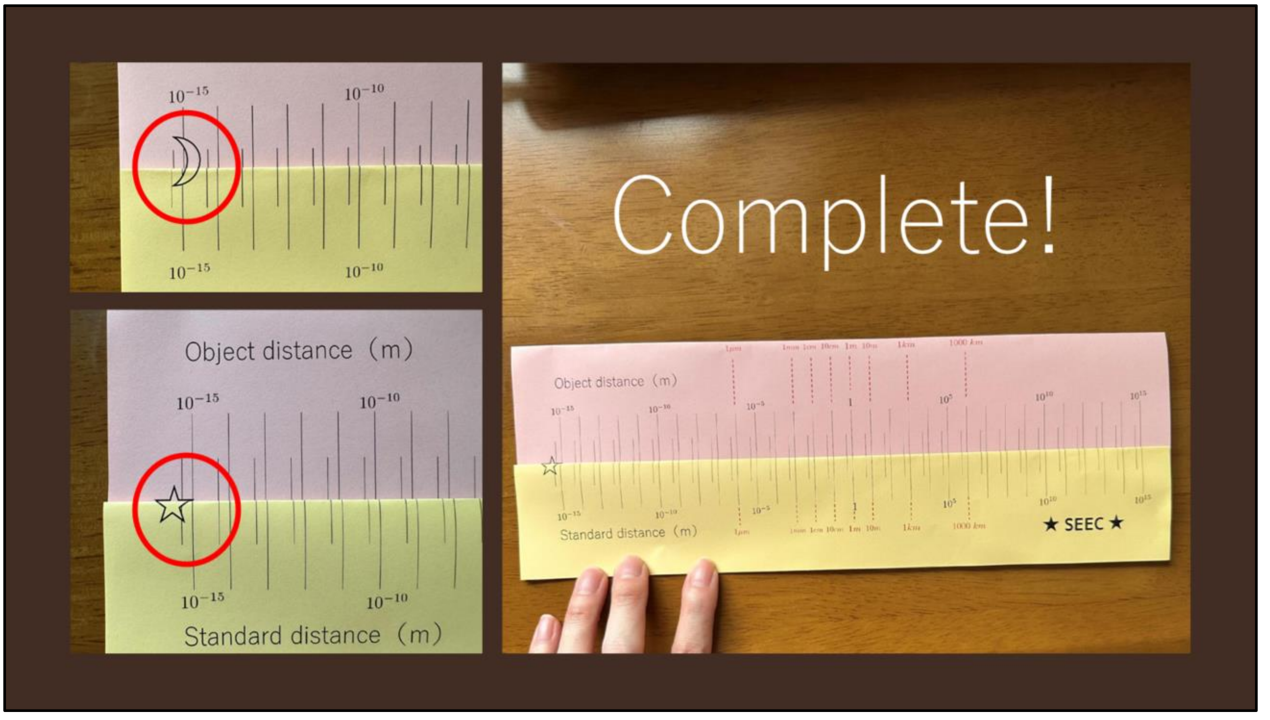


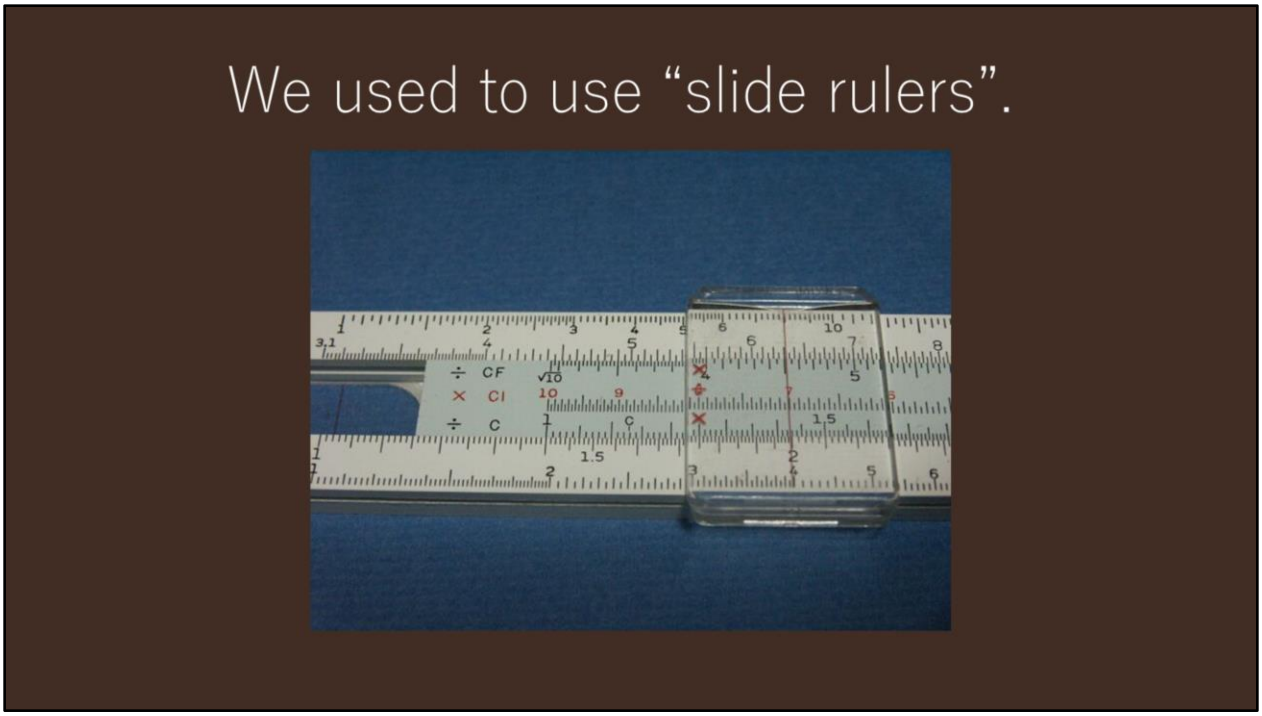
Fold it in half again please.

(Now, please fold only one side in half so you can see the ruler.
Turn it over, and do the same as the other side.)



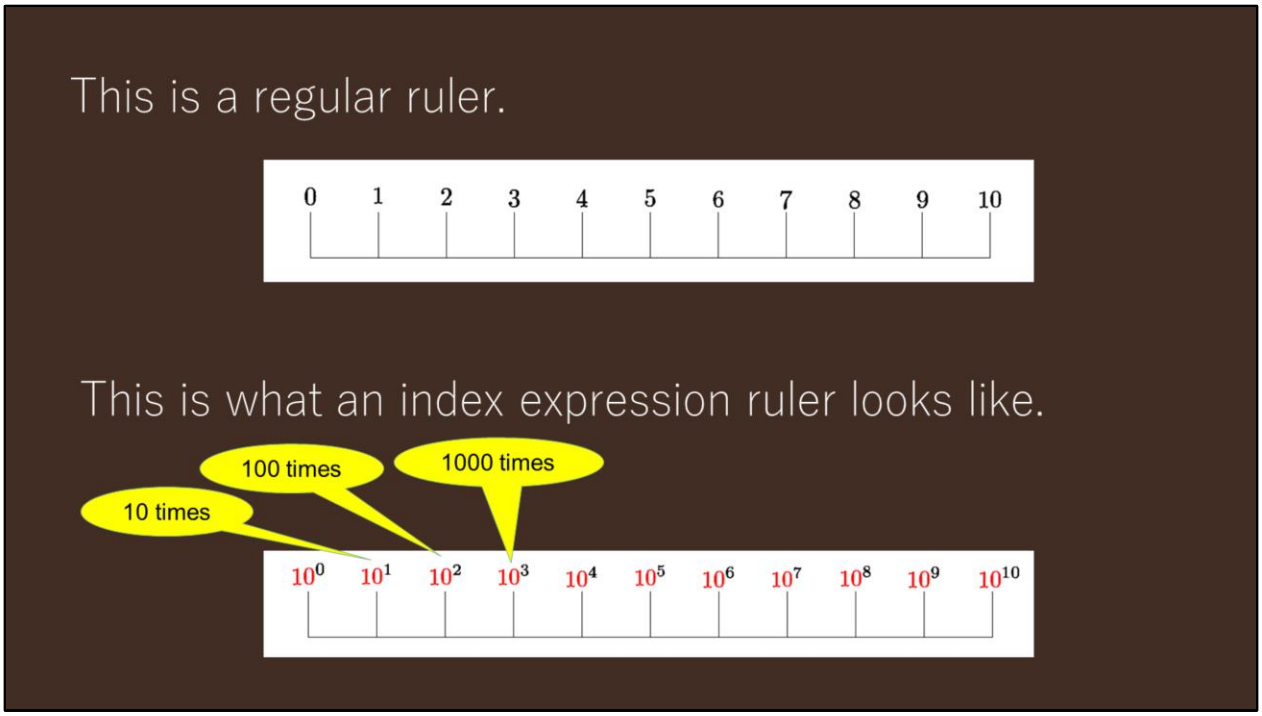
Now, please put the pink logarithm ruler inside the slot of the yellow logarithm ruler.

The small icon should be visible on the left side.
You should be able to see a moon on one side and a star on the other side?



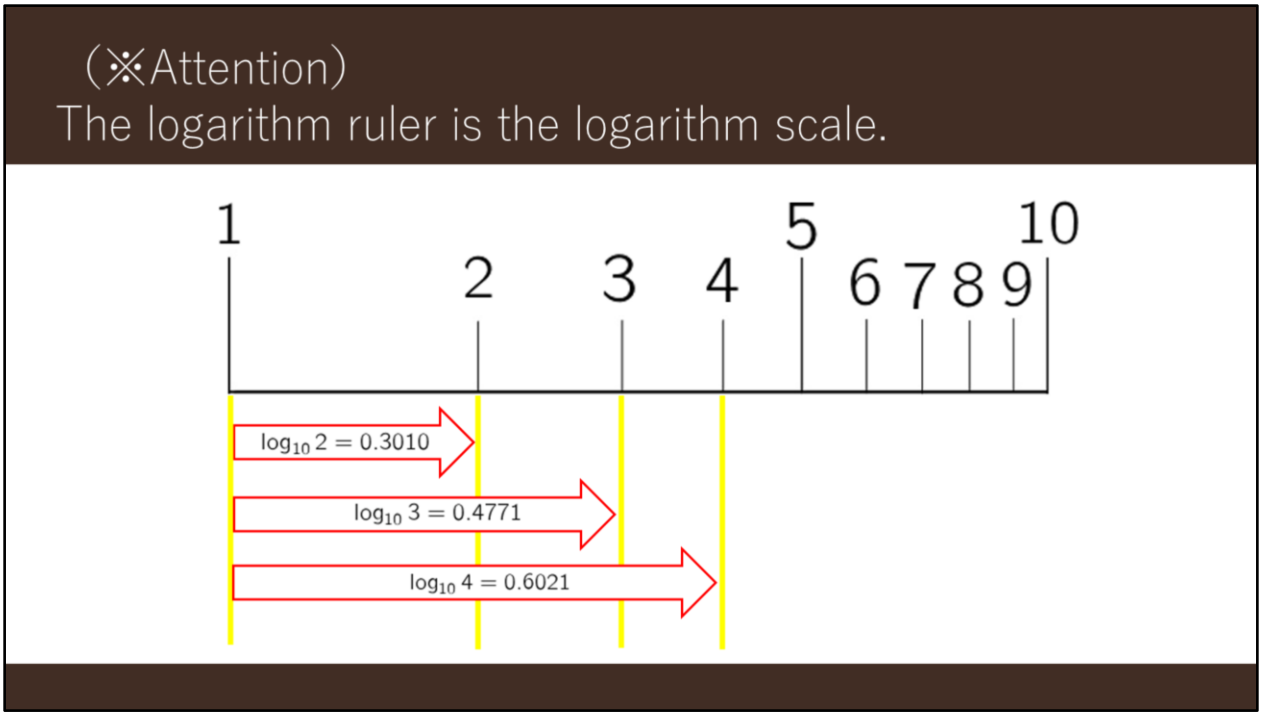
In the past, we actually used logarithm rulers like this.
Now, there are sold only at a net auction.

You have a paper version today.



The scale on a ruler is marked at equal distance.
(0,1,2,3・・・)

On your ruler you see the index expression.
The index expression starts from 1.
The scale on the right side reads 10^{1}, 10^{2}, 10^{3}.

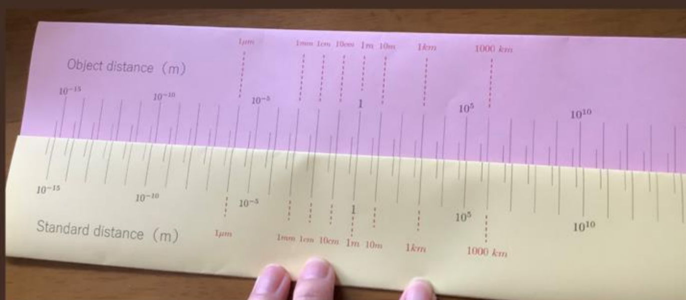


On the scale of the index expression, the scale between the numerical value was marked by the common natural logarithm.

The index expression and the logarithm expression have a deep connection.

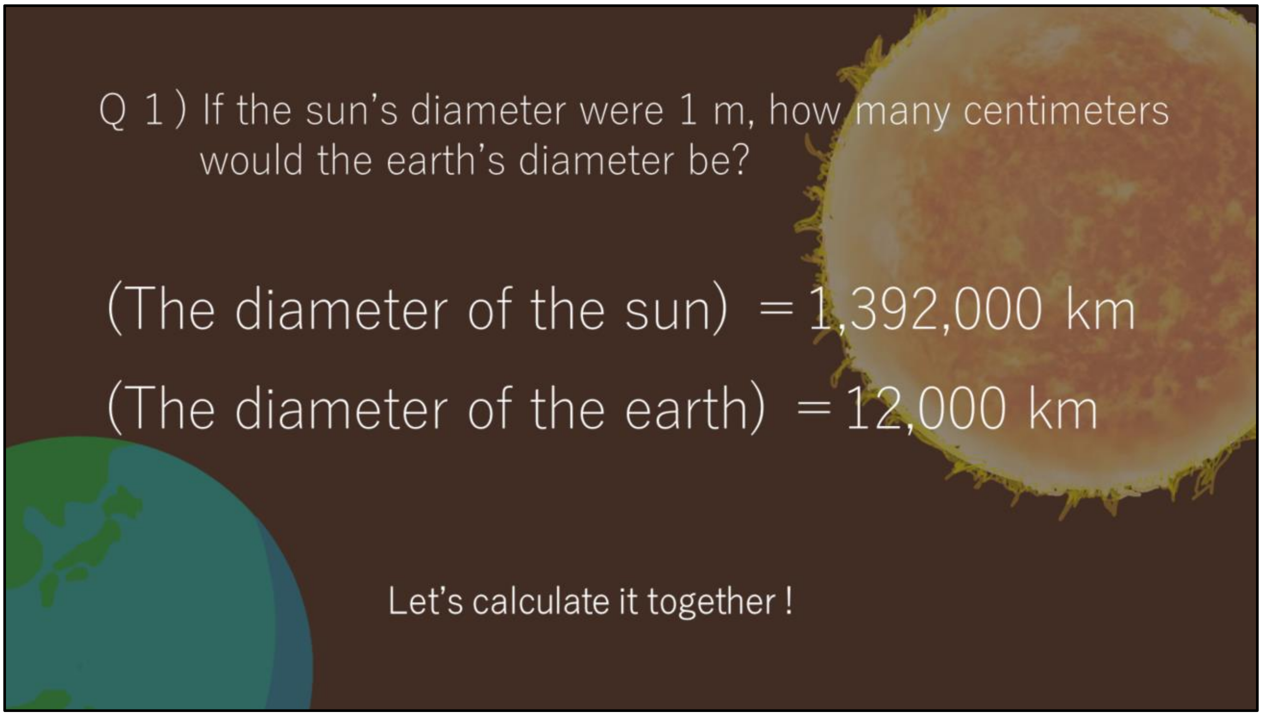
3 . Some examples with tools

Using the logarithm ruler,
let’s grasp the image of large and small numbers !



The logarithmic rule is useful for numbers that are too large or small to imagine.

Let’s look at some examples together.

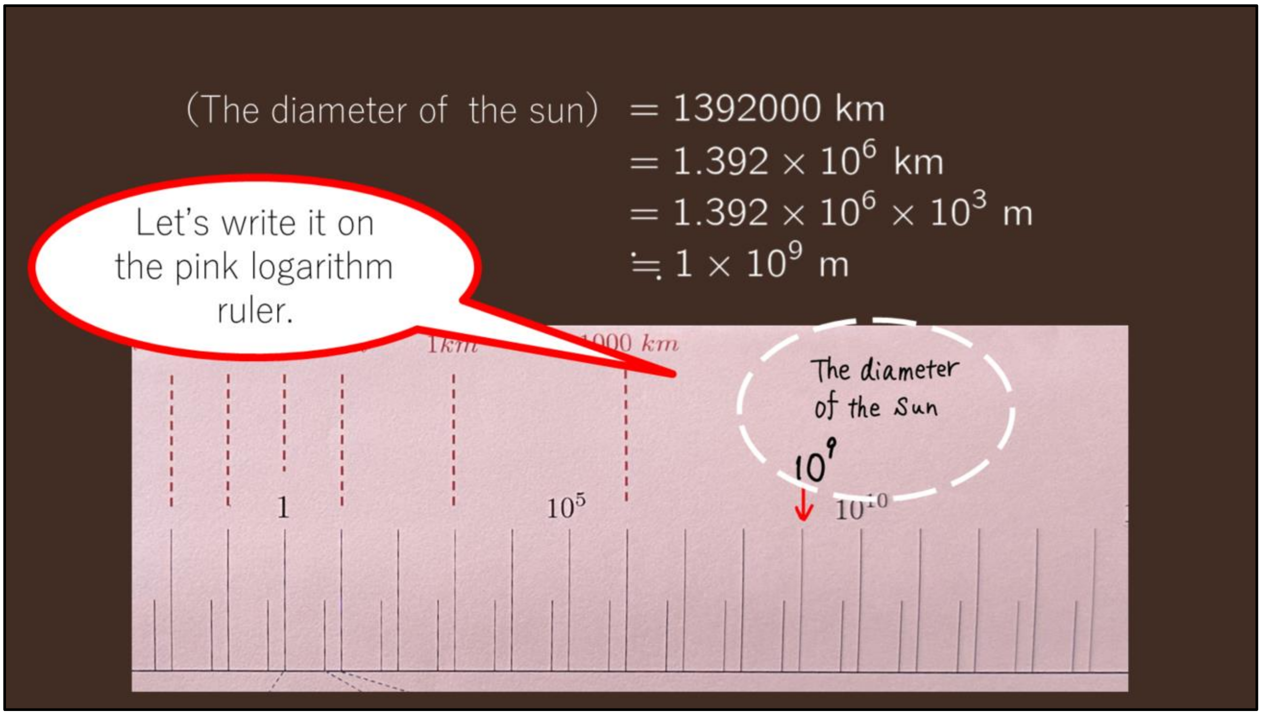


Question 1) If the sun's diameter were 1 m, how many centimeters would the earth's diameter be?

The diameter of the sun is 1,392,000 km.
The diameter of the earth is 12,000 km.

If the diameter of the sun becomes 1 m.
What would be the length of the diameter of the earth?

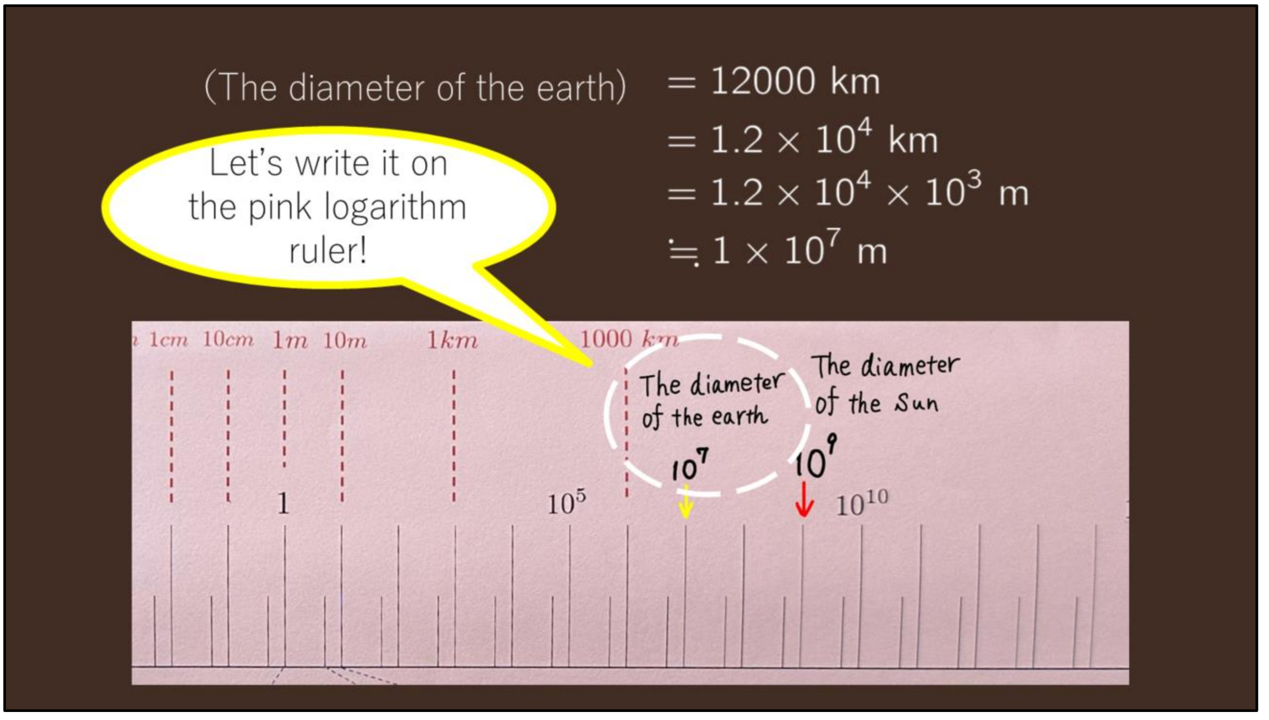
Let's use the logarithm ruler.



The actual diameter of the sun is 1,392,000 km.

Let's change "km" to "m".
You can approximate the digits.
The result would be about 10^{9} m.

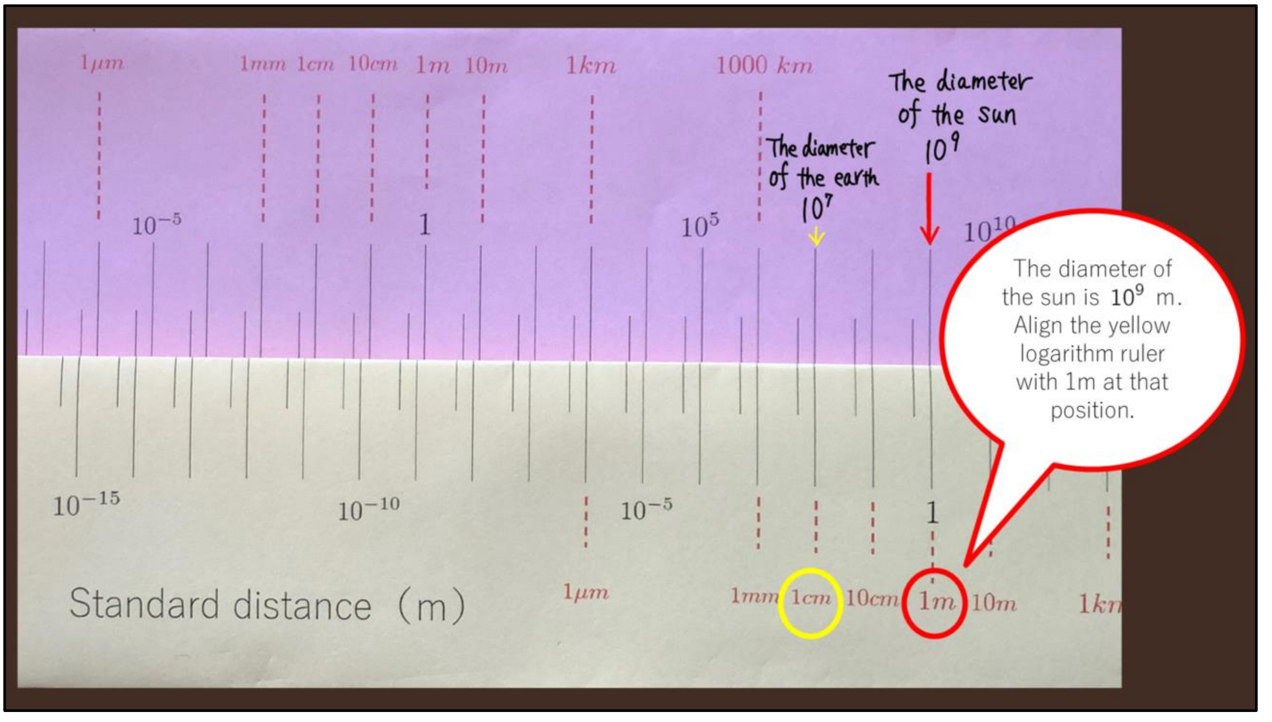
At the 10^{9} on the pink logarithm ruler,
Please write "The diameter of the sun".
Also, a simple mark is fine.



Next, the actual diameter of the earth is 12,000 km.

Let's change "km" to "m" as previously done.
The result would be about 10^{7} m.

At the 10^{7} on the pink logarithm ruler,
Please write "The diameter of the earth".



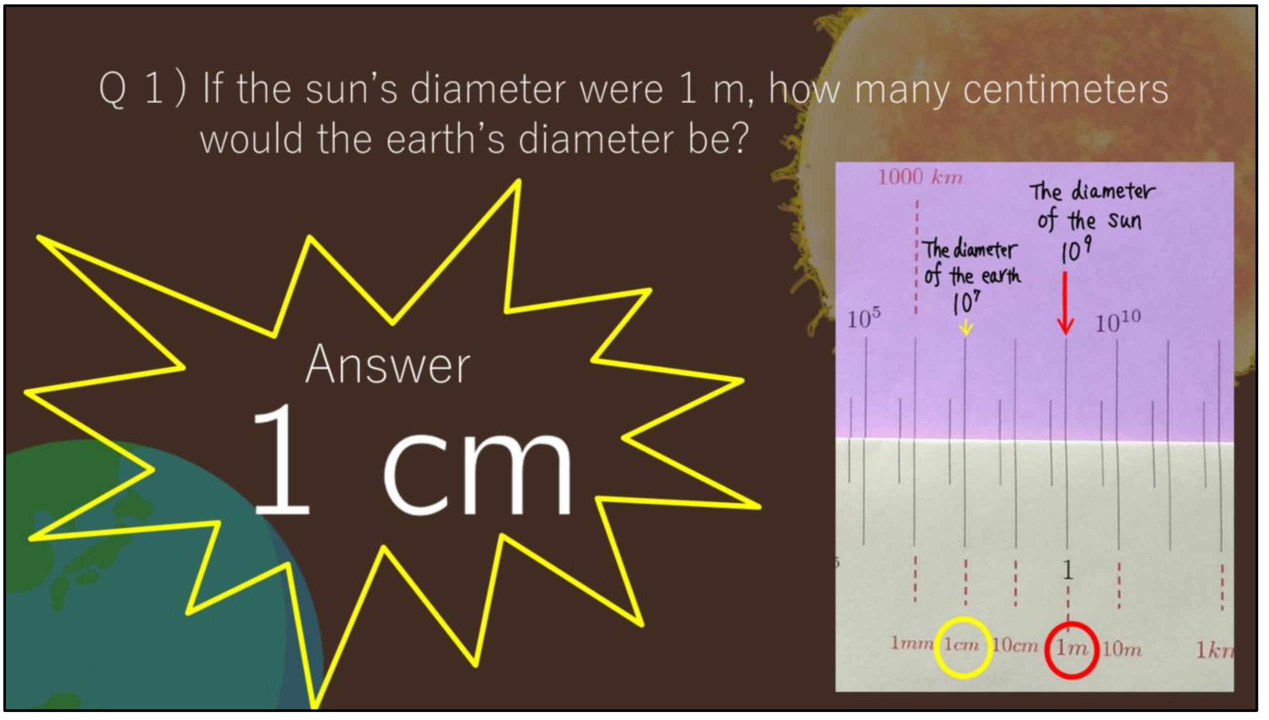
Now, the pink logarithm ruler should be put inside the yellow logarithm ruler slot.
Just like this.

The diameter of the sun is 10^{9} m.
Previously, you have made the assumption that the diameter of the sun were 1 m.

Therefore, you should adjust the mark at 1 m on the yellow logarithm ruler to the mark of 10^{9} m on the pink logarithm ruler.

At this time, let’s look at the diameter of the earth as shown at the 10^{7} m on the yellow logarithm ruler.

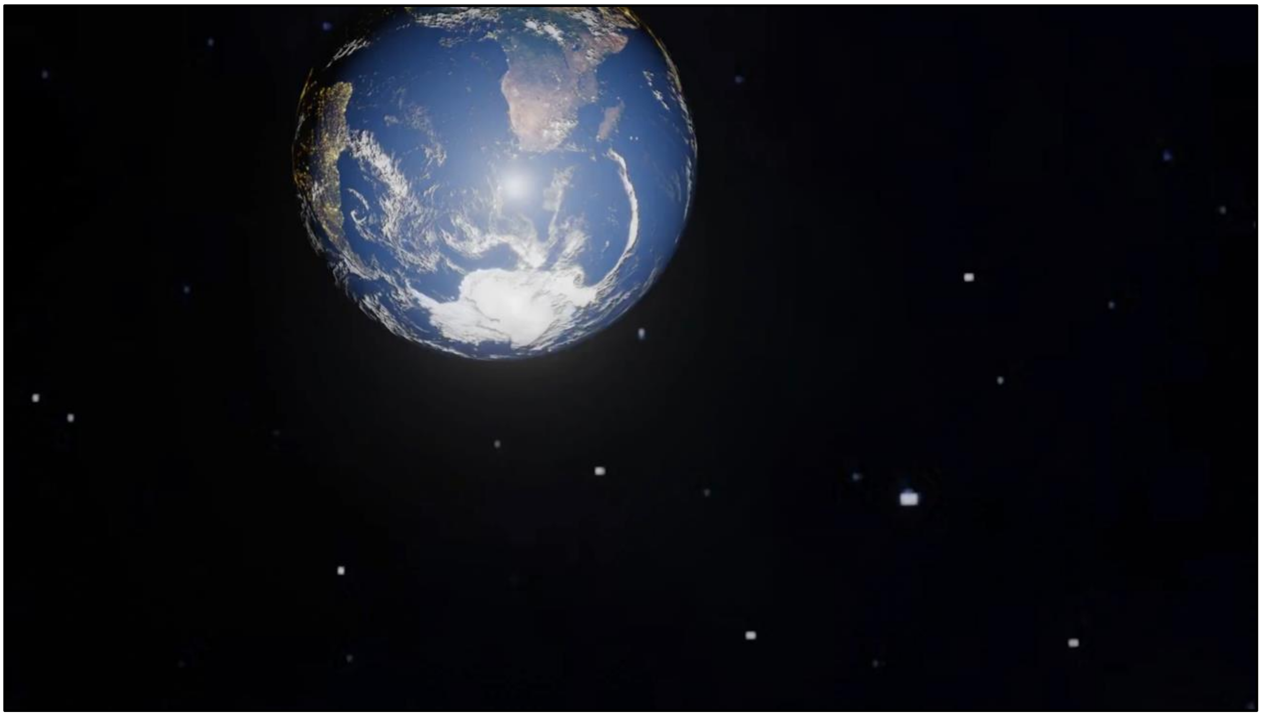
Did you get the answer?



Answer, the diameter of the earth would be about 1 cm.

We live on the earth.
The earth is very small compared to the sun.

Let me show you the next movie.



I made a video to make it easier to imagine the size of the earth and the sun.
How many earths are there here?
The earth is one, two, three…I can't count.

When you think about the sun and the earth, you imagine that they are big, right?
However, if the diameter of the sun were to be 1 m, the diameter of the earth would be 1 cm.
It would look like this.

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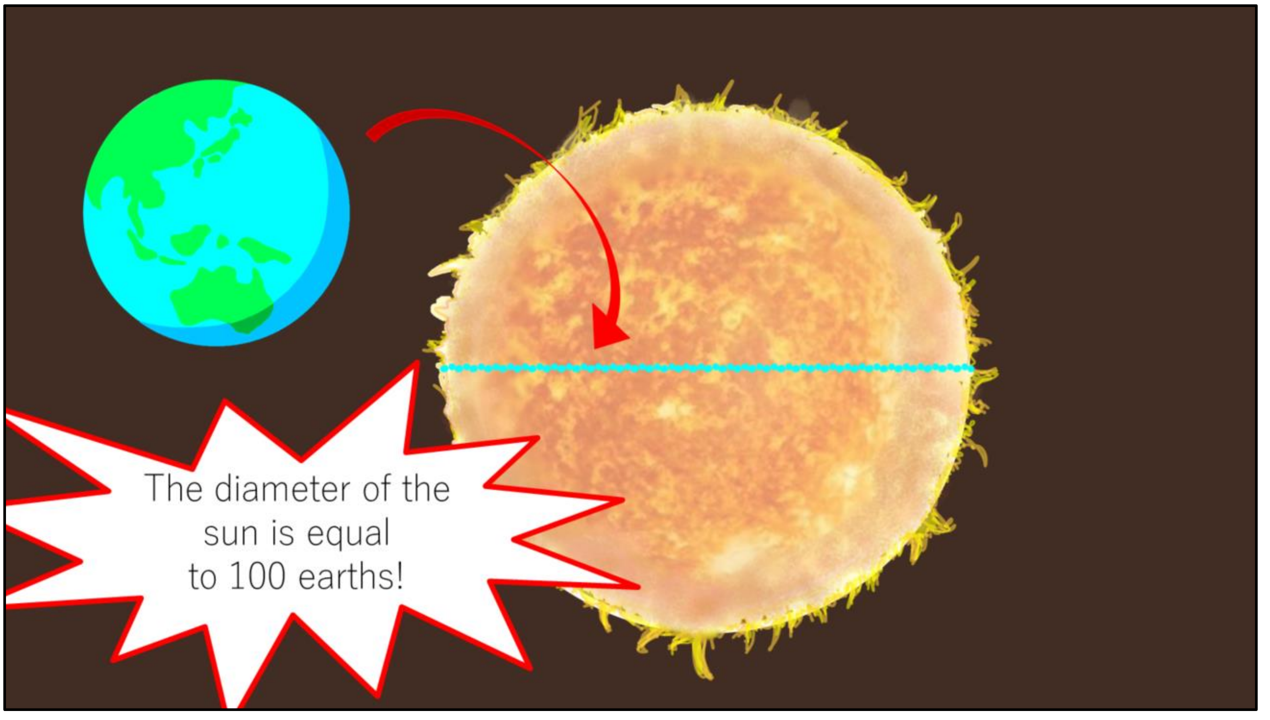
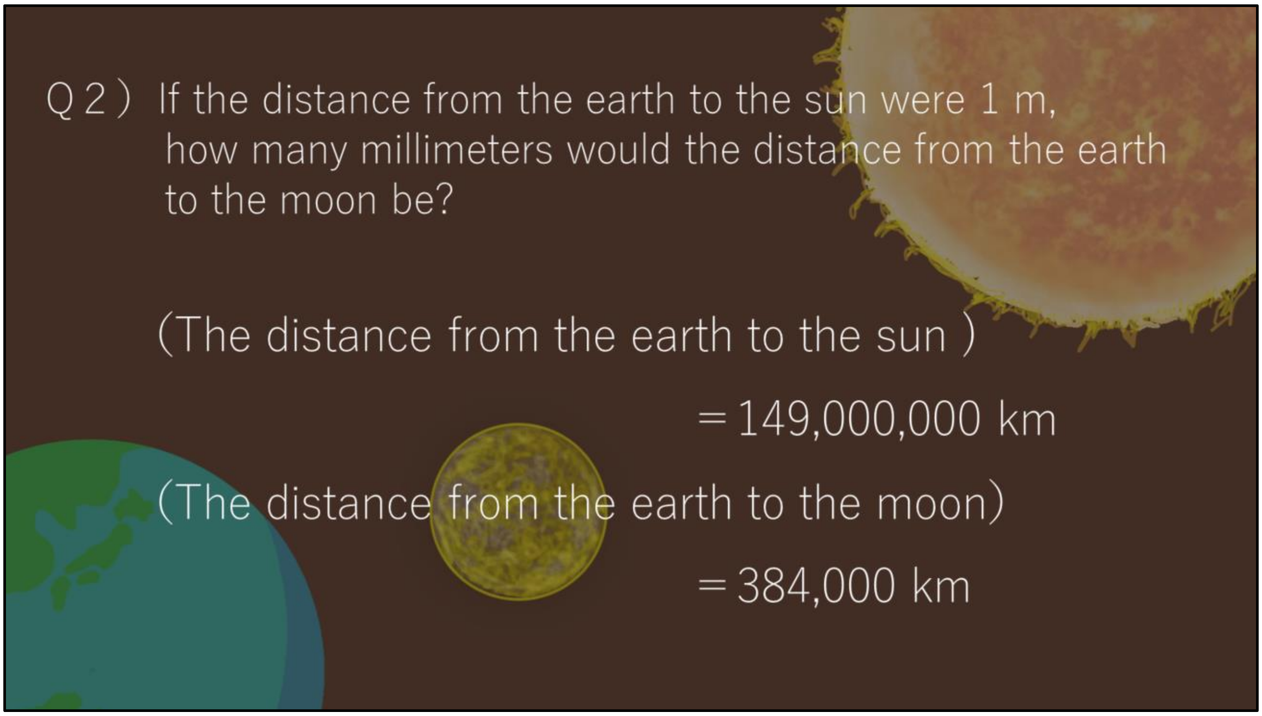


The diameter of the sun is the size of 100 earths.
The sun that brightens our days is 100 times larger than the earth.



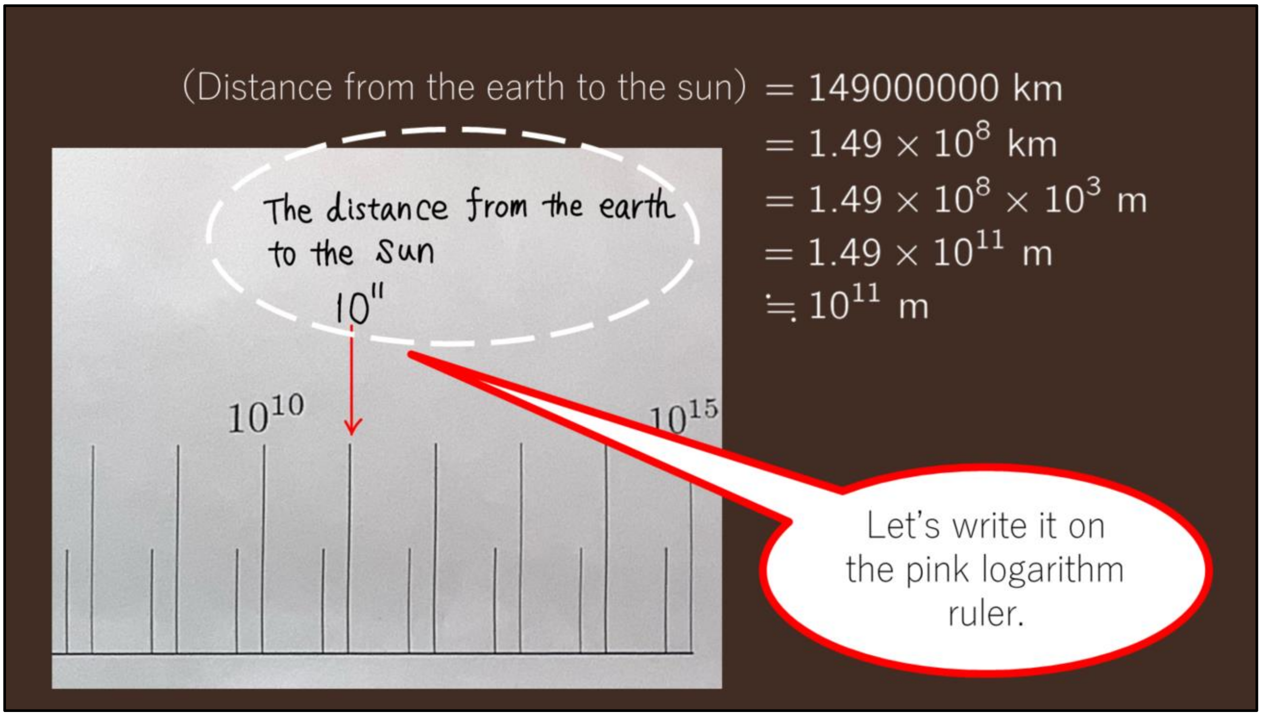
Let’s move on to the next question.

Question 2) If the distance from the earth to the sun were 1 m, how many millimeters would the distance from the earth to the moon be?

The distance from the earth to the sun is 149,000,000 km.
The distance from the earth to the moon is 384,000 km.

If the distance from the earth to the sun would be 1 m, we wouldn’t be able to live on the earth.
Question: If the distance from the earth to the sun were 1 m, how far would the earth be from the moon?

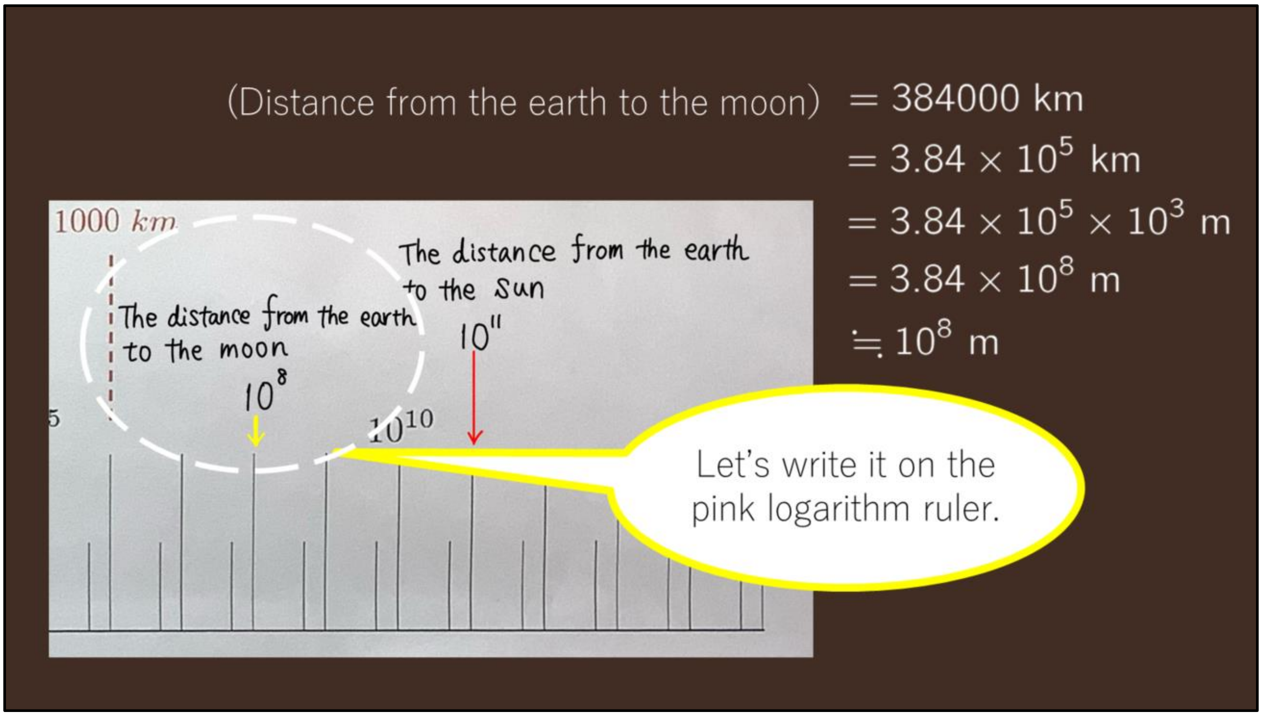
Let’s use the logarithm ruler.



The actual distance from the earth to the sun is 149,000,000 km.

Let's change "km" to "m" as previously done in question 1.
The result would be about 10^{11} m.

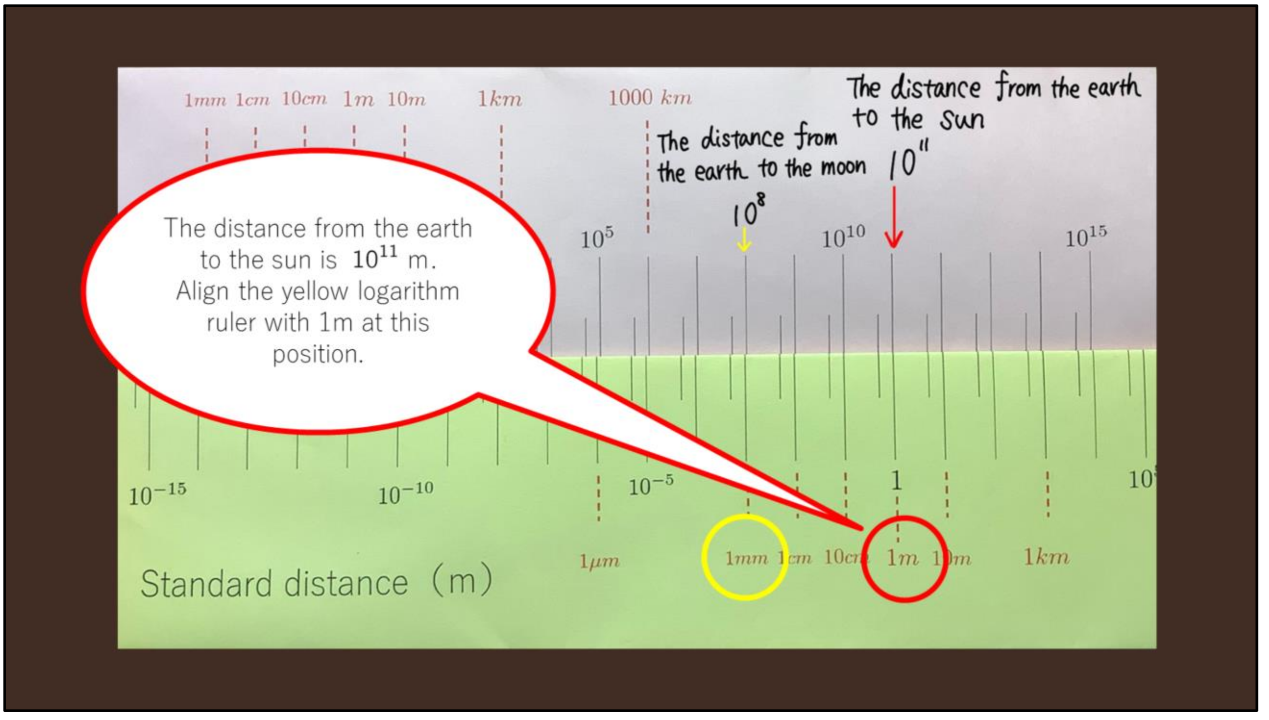
At the 10^{11} on the pink logarithm ruler, please write "The distance from the earth to the sun".
Also, a simple mark is fine.



Next, the distance from the earth to the moon is 384,000 km.

Let’s change “km” to “m” as previously done.
The result would be about 10^{8} m.

At the 10^{8} on the pink logarithm ruler, please write “The distance from the earth to the moon ”.



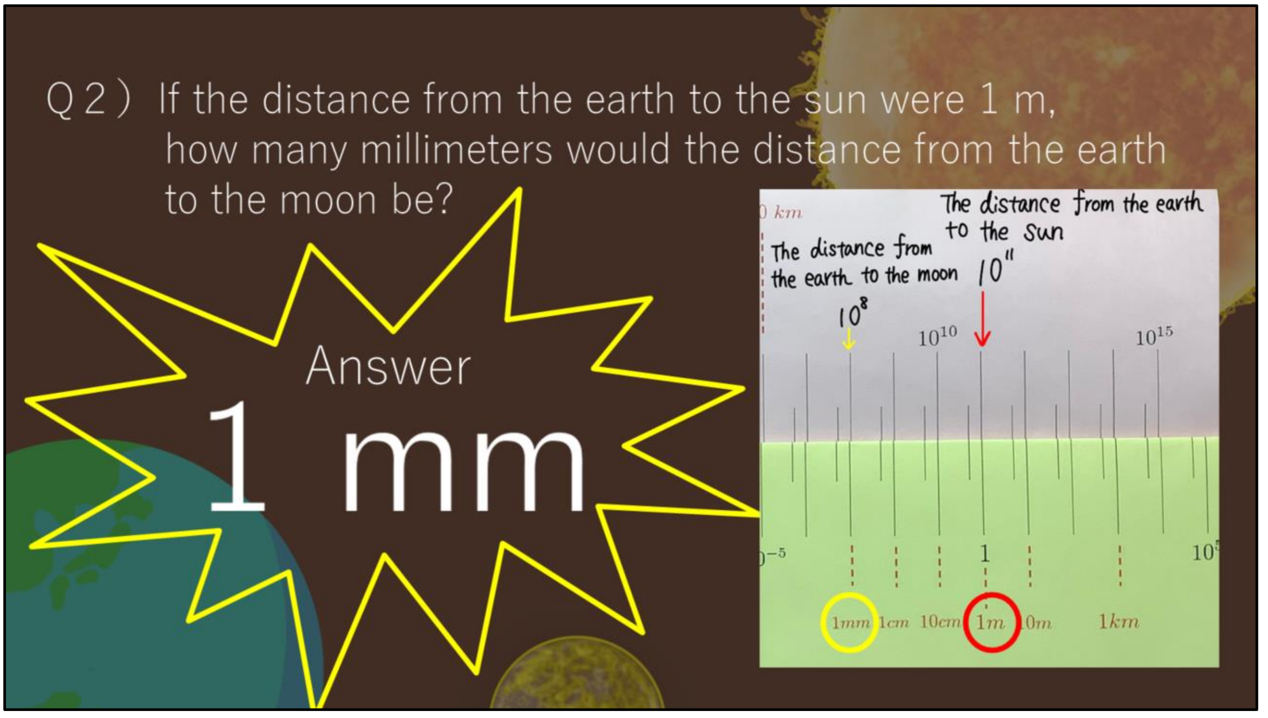
Now, the pink logarithm ruler should be put inside the yellow logarithm ruler slot.

The distance from the earth to the sun is 10^{11} m.
Previously, you have made the assumption that the distance from the earth to the sun were 1 m.

Therefore, you should adjust the mark at 1m on the yellow logarithm ruler to the mark of 10^{11} m on the pink logarithm ruler.

At this time, let’s look at the distance from the earth to the moon as shown at the 10^{8} m on the yellow logarithm ruler.

Did you get the answer?



The distance from the earth to the moon was about 1 mm.

Let me show you the next movie.

I made a video to make it easier to imagine the distance between the earth, the sun and the moon.
I made it with an application software “Blender”.
The small ball seen near the earth is the moon.

If the distance from the earth to the sun were 1 m, the distance from the earth to the moon would be 1 mm.

The earth and the moon would be very close.

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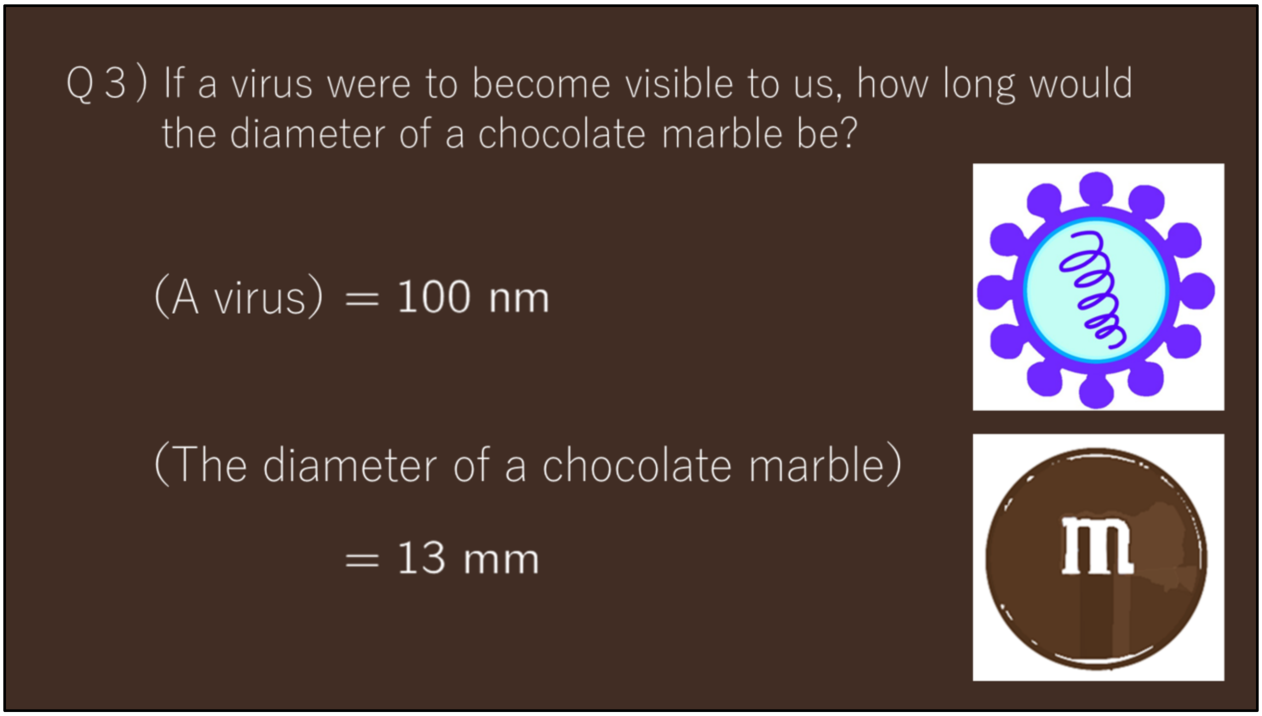
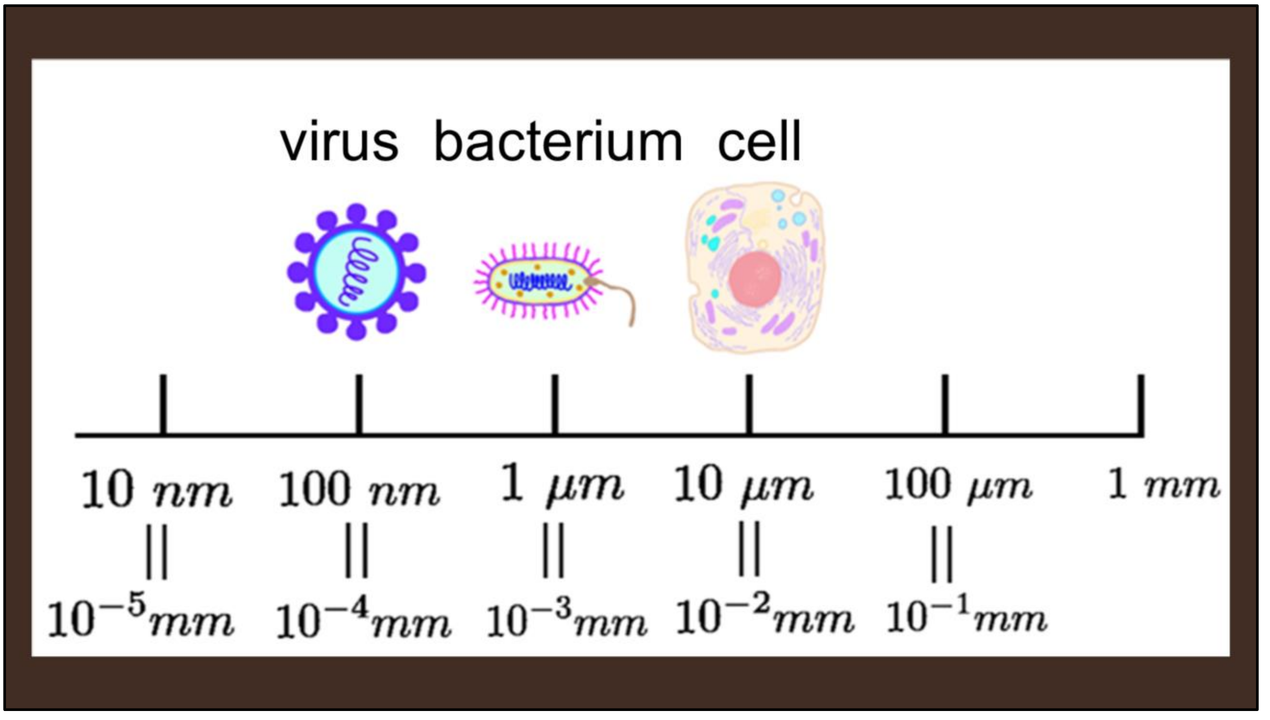


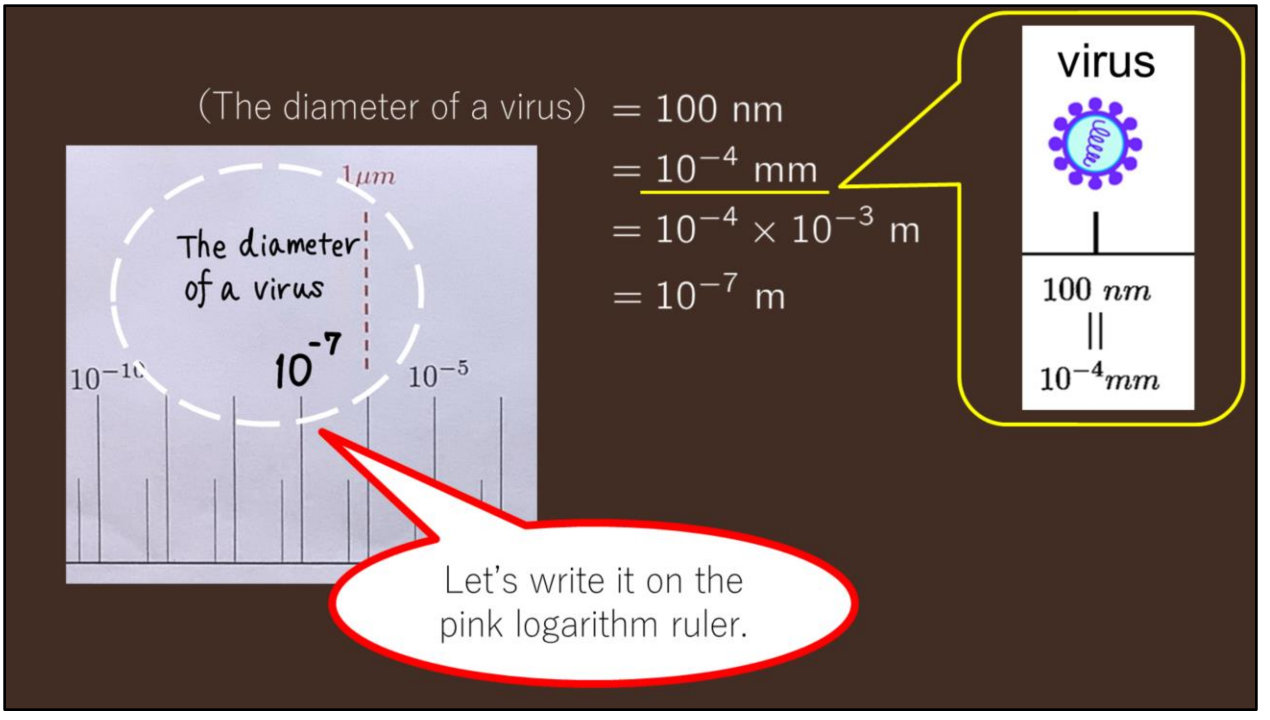
Now let’s grasp the concept of small numbers.

Question 3) If a virus were to become visible to us, how long would the diameter of a chocolate marble be?

Let’s compare a chocolate marble and a virus, assuming that a virus is visible to the human eye.
The actual diameter of a virus is 100 nm and the diameter of a chocolate marble is 13 mm.



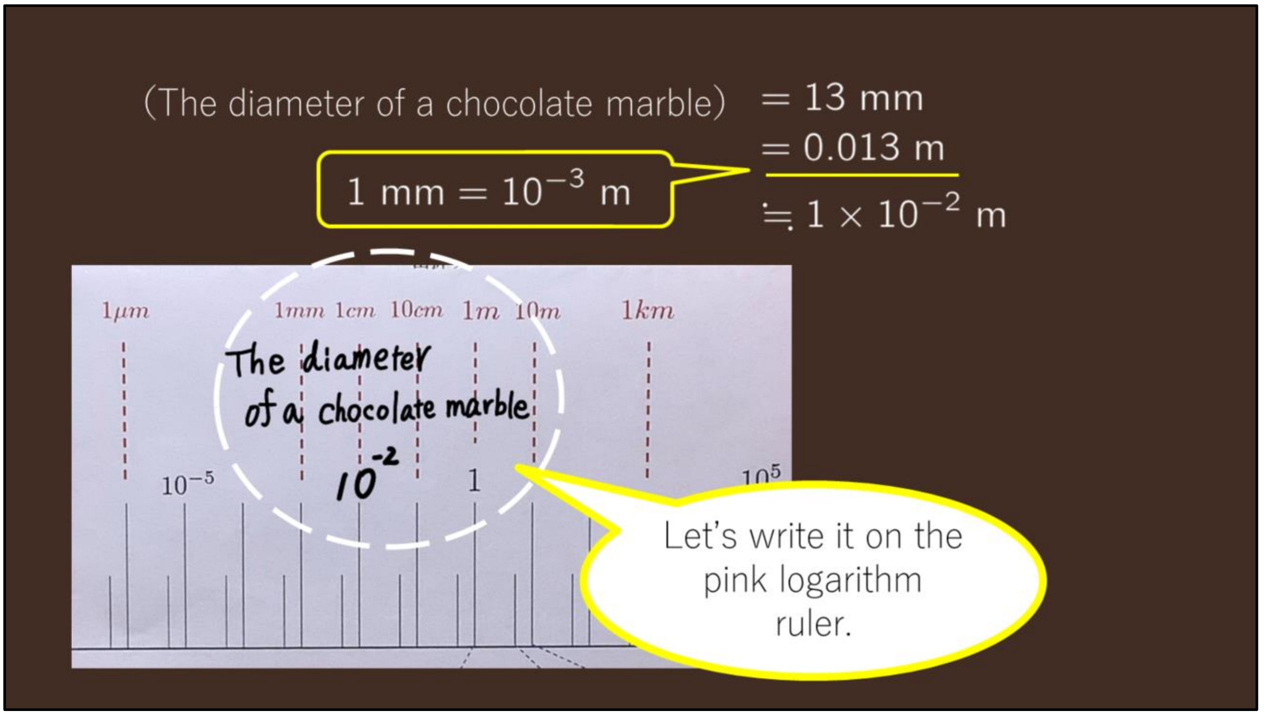
Please look at this figure.
A virus is smaller than the bacterium and the cell.



The actual diameter of a virus is 100 nm.

Let's change "nm" to "m" as previously done.
The result would be about 10^{-7} m.

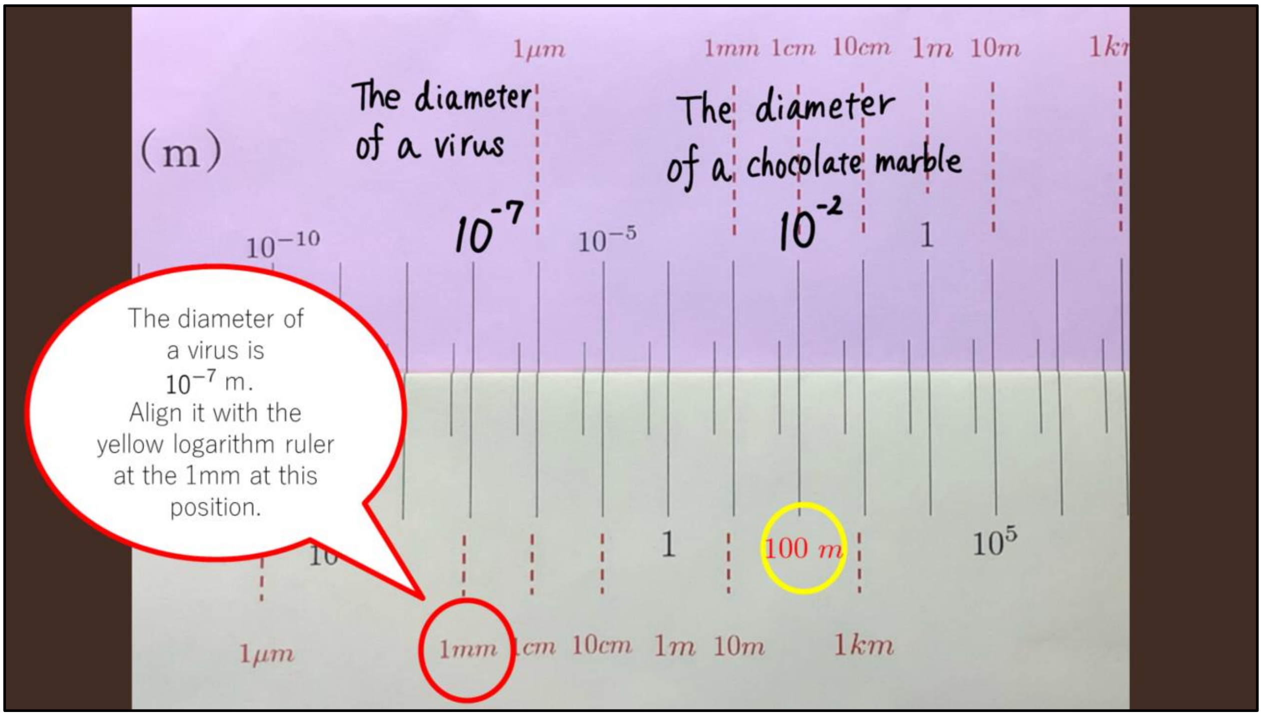
At the 10^{-7} on the pink logarithm ruler, please write "The diameter of a virus".
Also, a simple mark is fine.



The actual diameter of a chocolate marble is 13 mm.

Let's change “mm” to “m” as previously done.
The result would be about 10^{-2} m.

At the 10^{-2} on the pink logarithm ruler, please write “The diameter of a chocolate marble”.



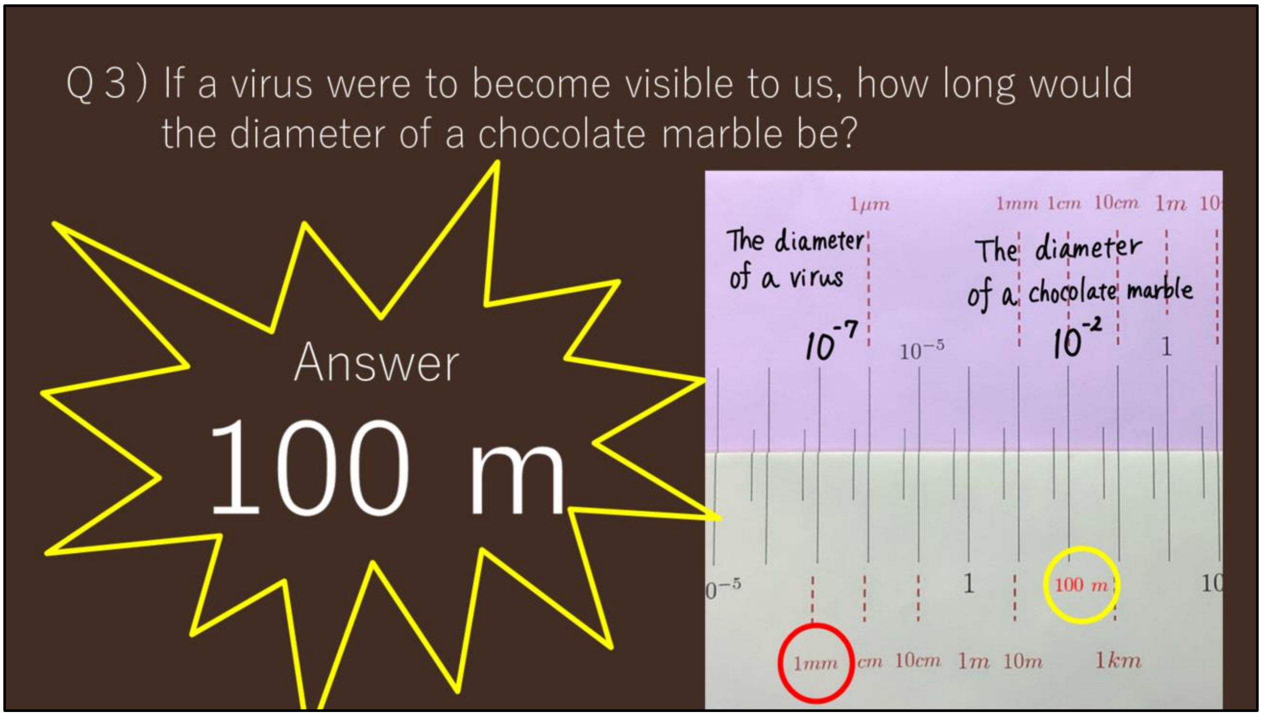
Now, the pink logarithm ruler should be put inside the yellow logarithm ruler slot.

The diameter of a virus is 10^{-7} m.
Previously, we assumed that a virus is visible to the human eye.

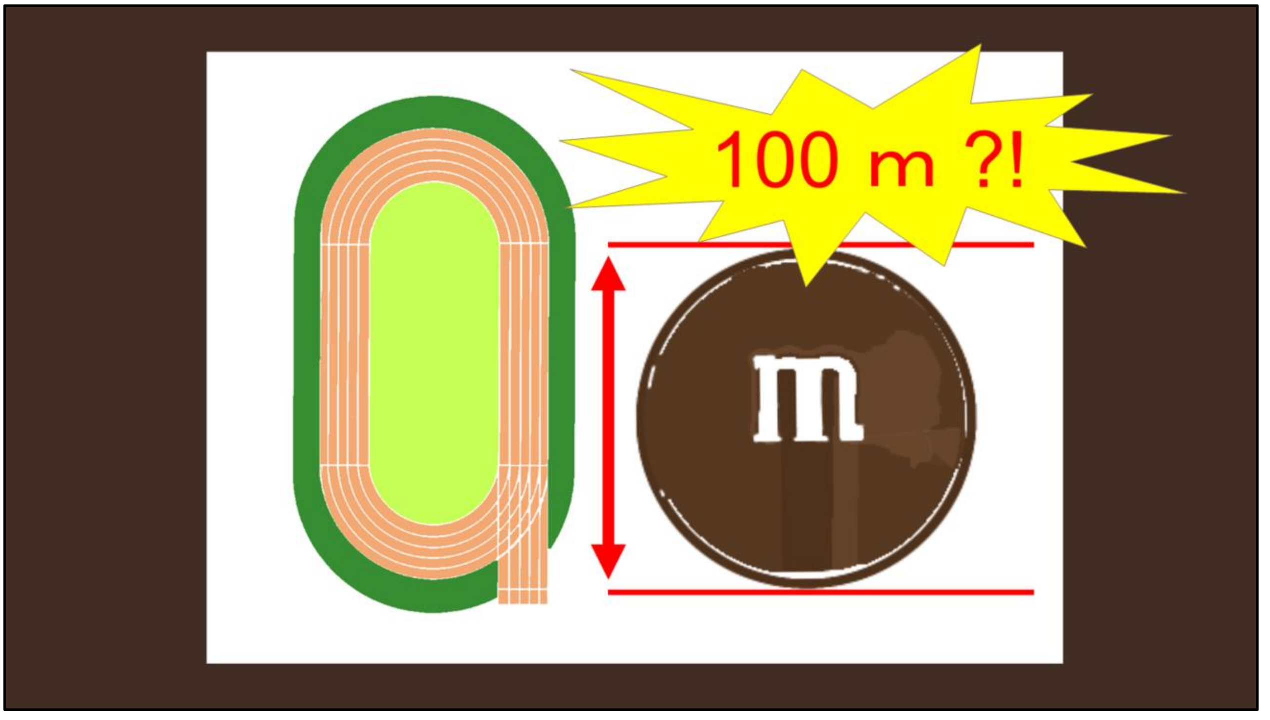
Therefore, you should adjust the mark at 1 mm on the yellow logarithm ruler to the mark of 10^{-7} m on the pink logarithm ruler.

At this time, let’s look at the diameter of a chocolate as shown at the 10^{-2} m on the yellow logarithm ruler.

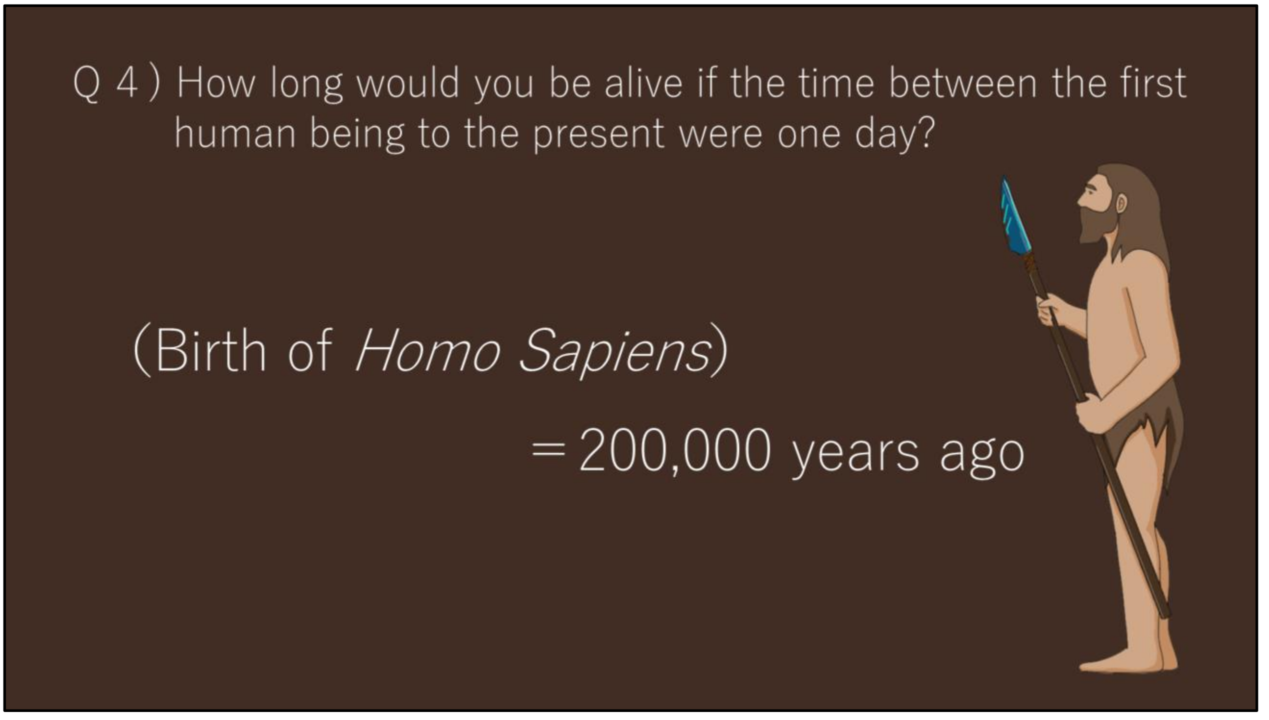
Did you get the answer?



The diameter of a chocolate marble will be 100 m.
When the diameter of a virus becomes about 1 mm, we can finally see a virus.



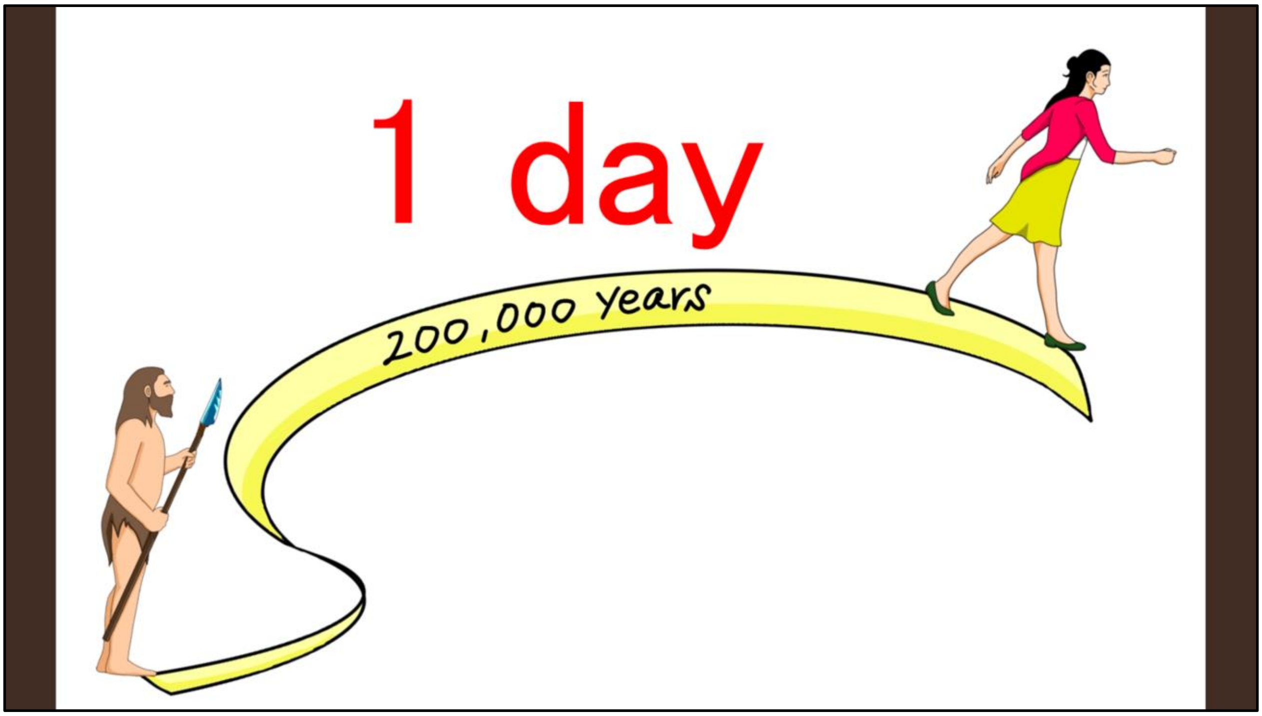
How can one chocolate be 100 m in diameter!?
How nice it would be to have a chocolate marble as big as 100 m!
By the way, astronauts eat a chocolate marble in the space!



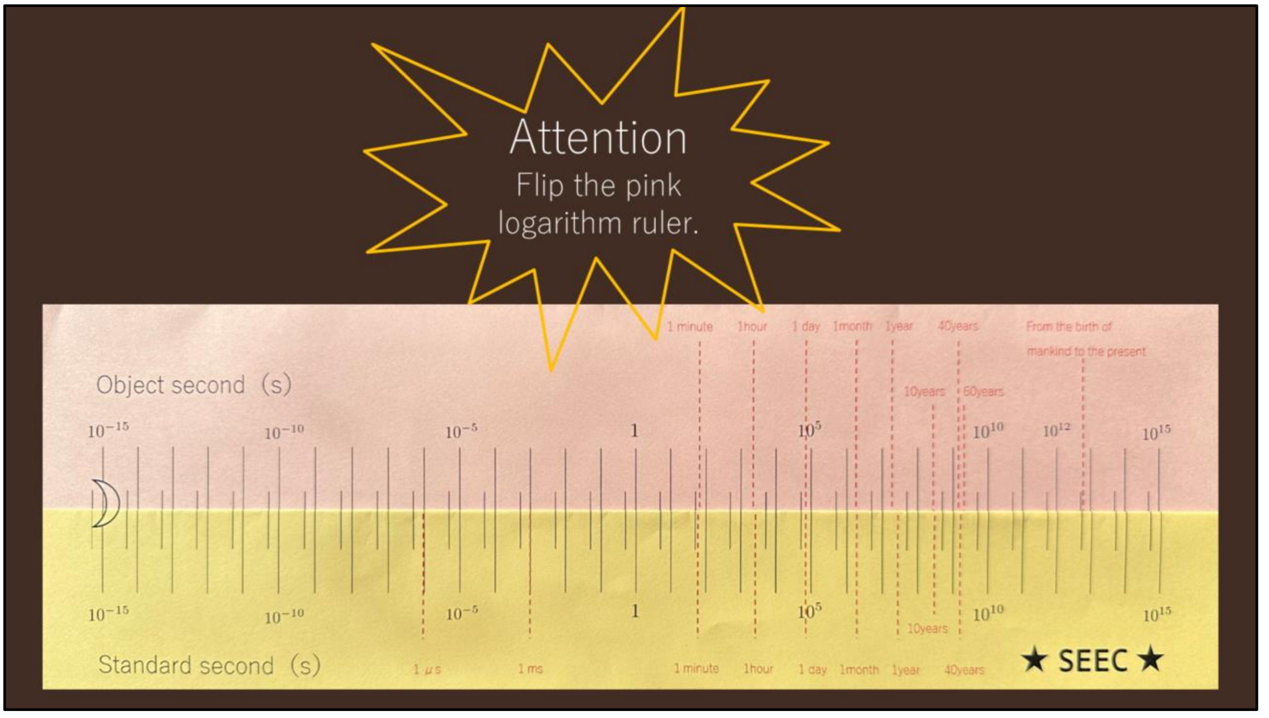
Last question!

How long would you be alive if the time between the first human being to the present were one day?

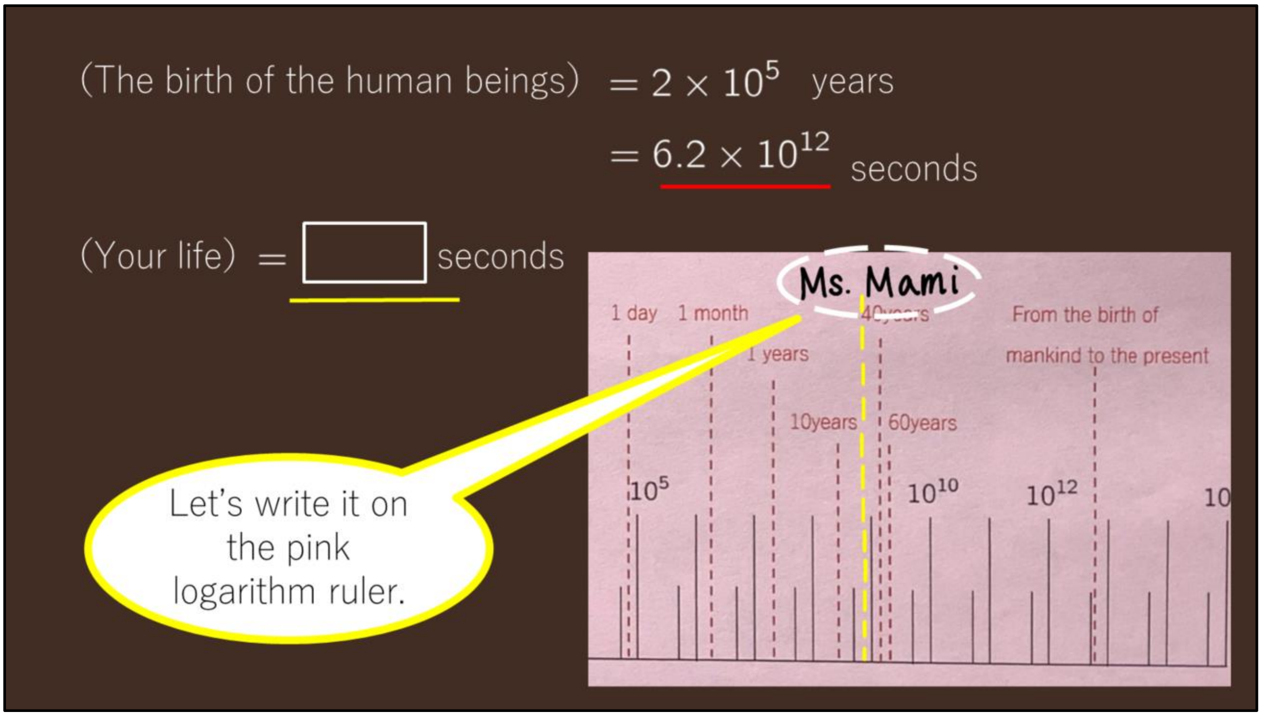
Please keep in mind that the birth of human beings was two hundred thousand years ago.



Let’s imagine that the human history(200,000 years) from the birth of the first human to the present is one day.



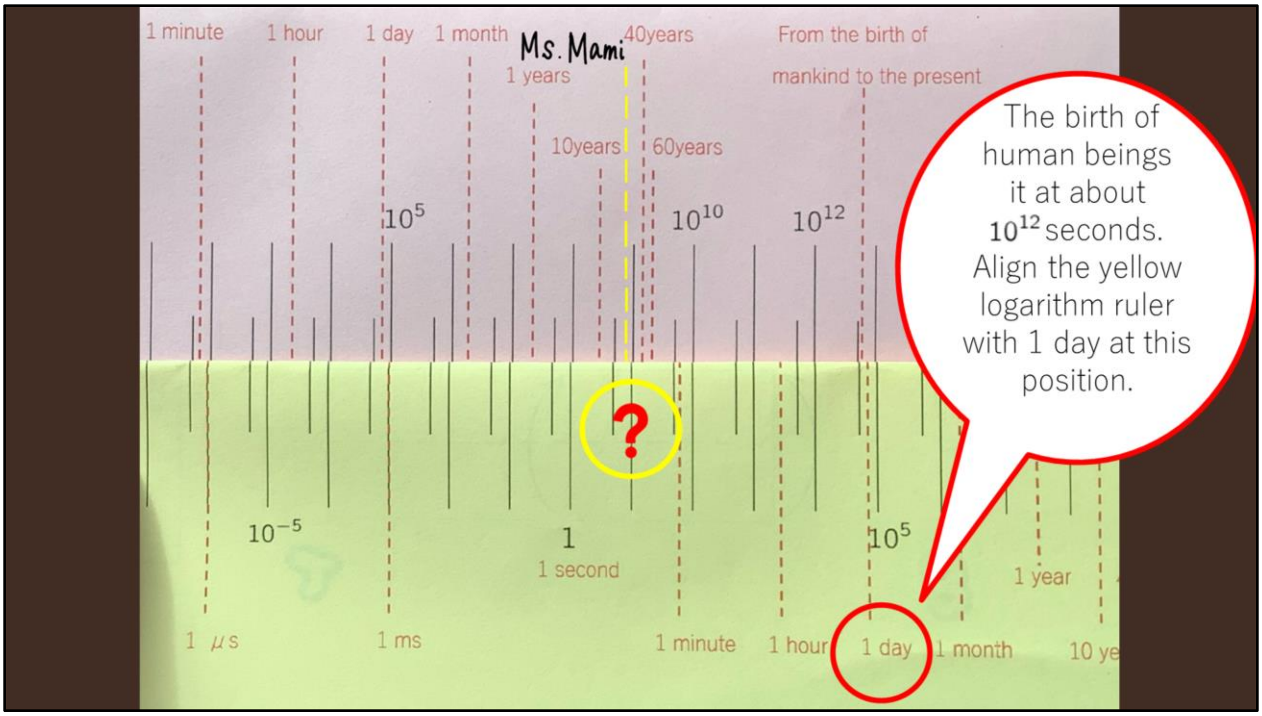
Let’s use the ruler here again.
Flip the logarithm ruler turn around.
Do you see the mark of the moon on the left side?



The birth of the first human being was two hundred thousand years ago.

Now, we should calculate our ages in seconds.
But it's a lot of calculation!
So, for your convenience, years are already written on the pink logarithm ruler.
Here! 10 years, 40 years and 60 years.
By the way, I'm here. 30 years old.

Please write your age.
If you are younger than me, please write "my life" to the left side of "Ms. Mami".
If you are older than me, please write "my life" to the right side of "Ms. Mami".

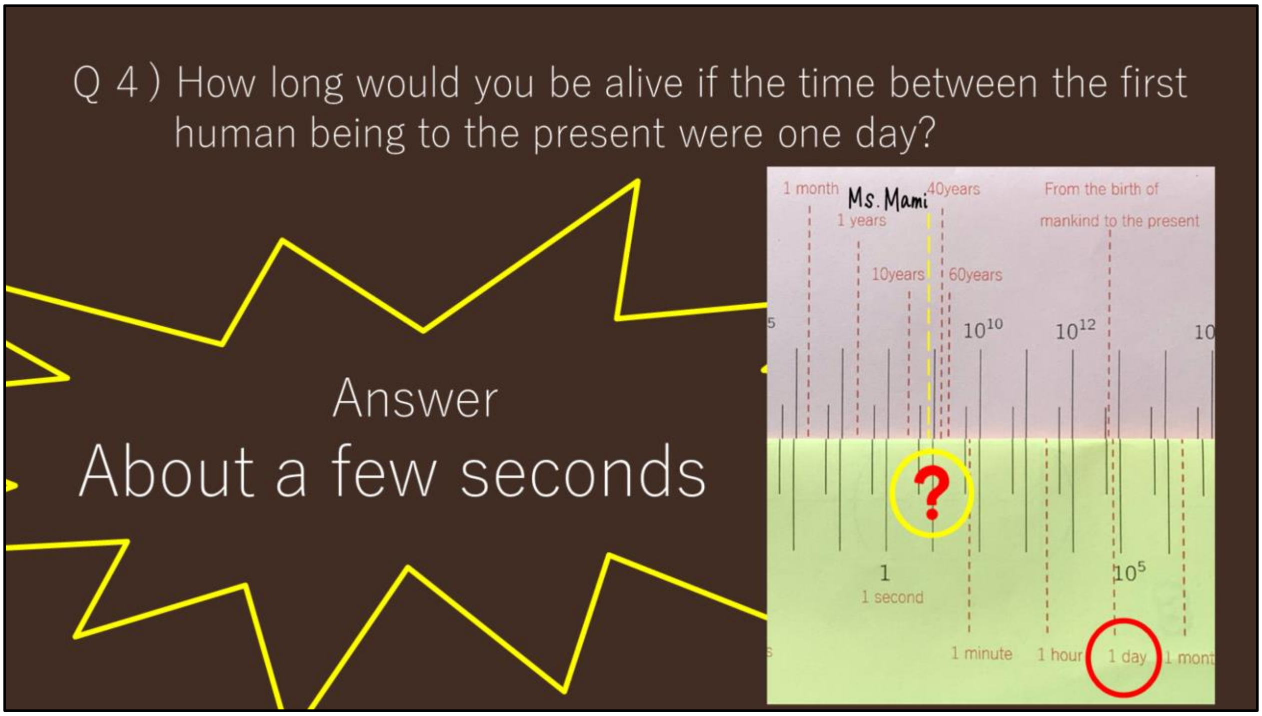


Now, the pink logarithm ruler should be put inside the yellow logarithm ruler slot.
The birth of human beings is at about 10^{12} seconds on the pink logarithm ruler.
Previously, we assumed that the birth of mankind to the present were 1 day(about 10^{5}).

Therefore, you should adjust the mark at 1 day on the yellow logarithm ruler to the mark of about 10^{12} seconds on the pink logarithm ruler.

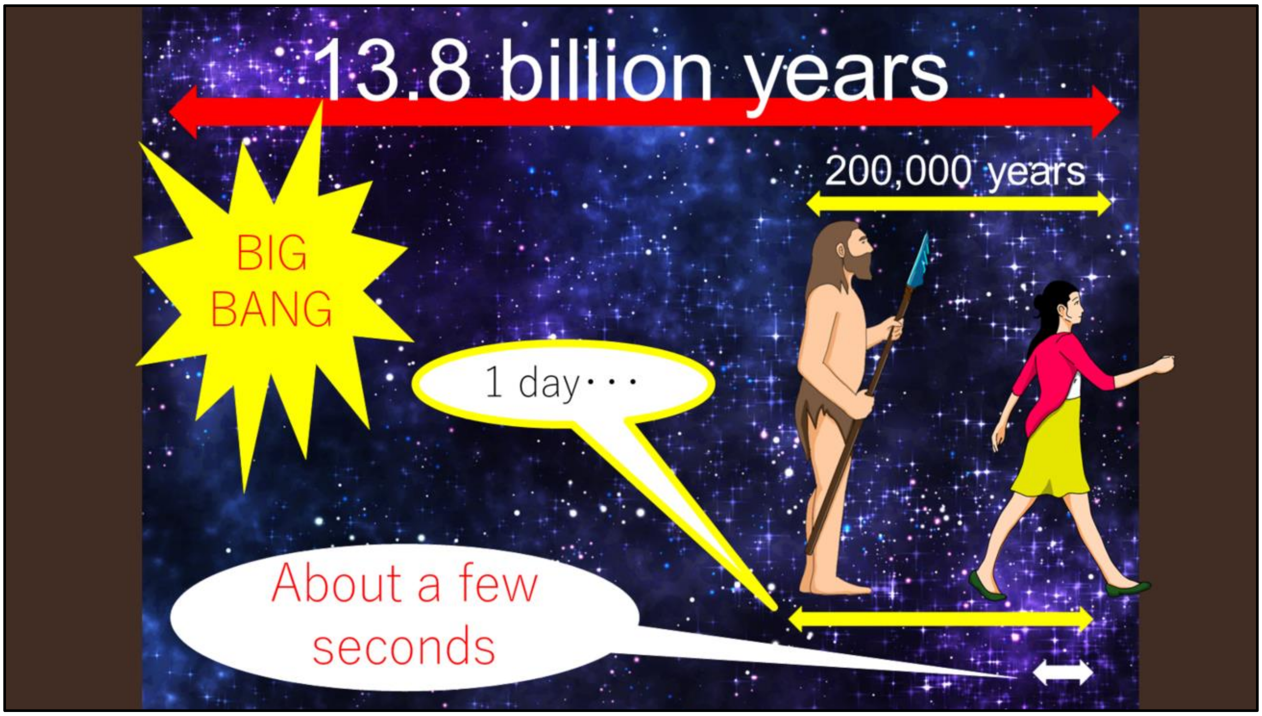
At this time, let’s look at the “my life” as shown at about a few seconds of the yellow logarithm ruler.

Did you get the answer?



The “my life” should be about a few seconds in the logarithm ruler.

If the history from the birth of the first human being to the present is 1 day, we are living for a very short period of time.



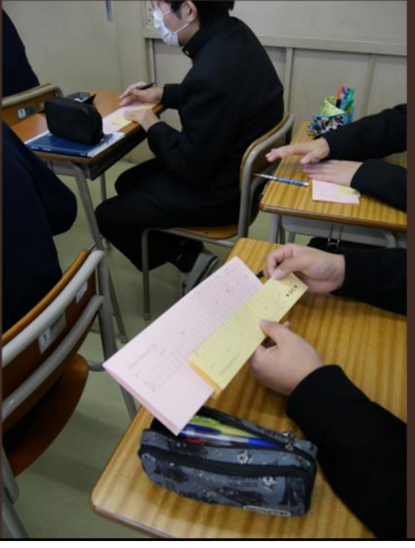
It has been 13.8 billion years since the universe came into existence.
If we assume that the period from the birth of our ancestors Homo Sapiens to the present were 1 day, we have been living for only about a few seconds.

On a cosmic scale, we only live a small part of the history.
It is very short, so I want to live a worthwhile life.

By the way, if we assume that the period from the birth of human beings to the present were 1 day, how long would it be from the start of the universe to the present day?

Think about it!

4 . The students' reactions



Here are my students.
Using these tools and this kind of calculation, they were able to imagine large and small objects.

In Japan, “m”, “cm”, and “mm” are the major units of length.
You can make a logarithmic scale with “ft” as the unit.

5 . Conclusion

(1) The students understood the usefulness of exponents and logarithms.

(2) The students could make quick comparison of the size of things with the logarithms ruler.

So, I will summarize today’s presentation.

First, the students understood the usefulness of exponents and logarithms.
Second, the students could make quick comparison of the size of things with the logarithms ruler.

That concludes my presentation.
Thank you so much for your interest and attention.

References and websites

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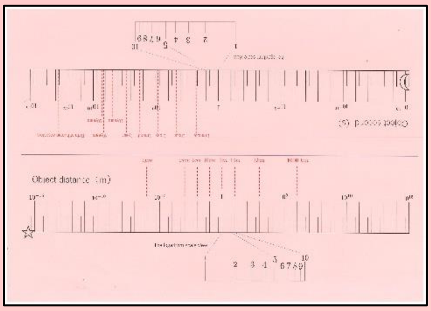
Here are the references for this presentation.

Application of Logarithms in Real Life

Mami KIKUCHI

Let's make a logarithm ruler together!

①



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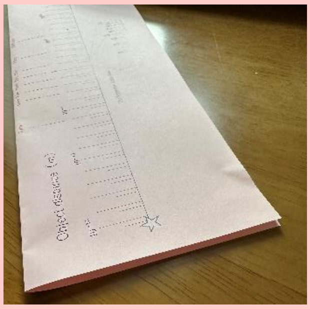
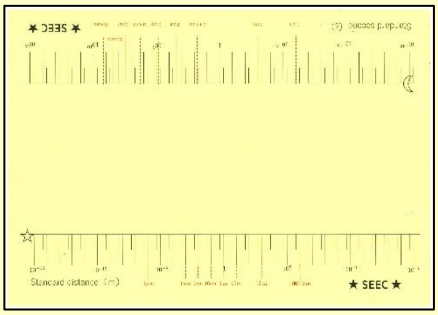
③



To make the pink logarithm ruler, please fold the paper in half.

①



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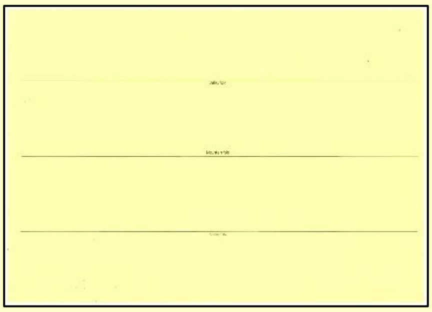

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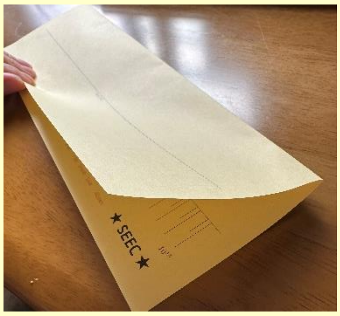

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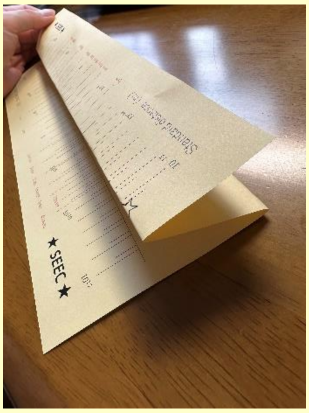
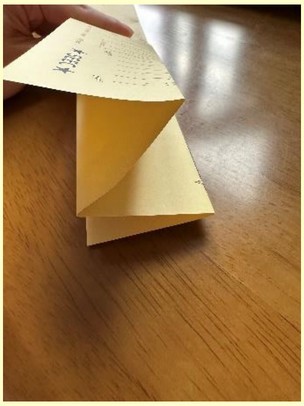


⑤

⑥



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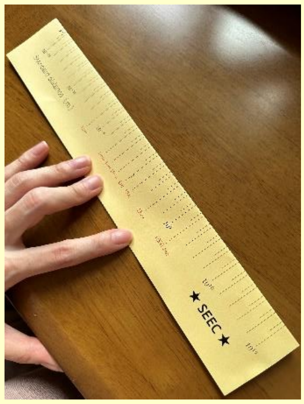
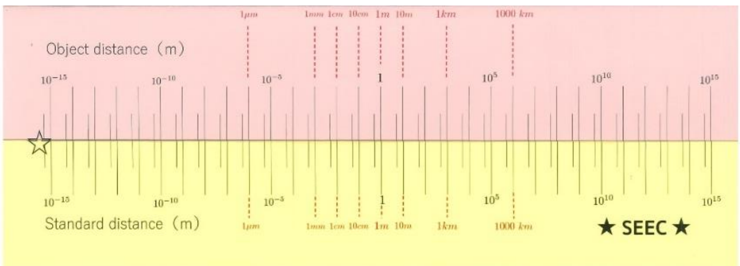


Next, Let's make a yellow logarithm ruler. In the same way, please fold it in half.

Now, the pink logarithm ruler should be put inside the yellow logarithm ruler slot. The mark should be visible on the left side that it is completed.

(Logarithm ruler →)



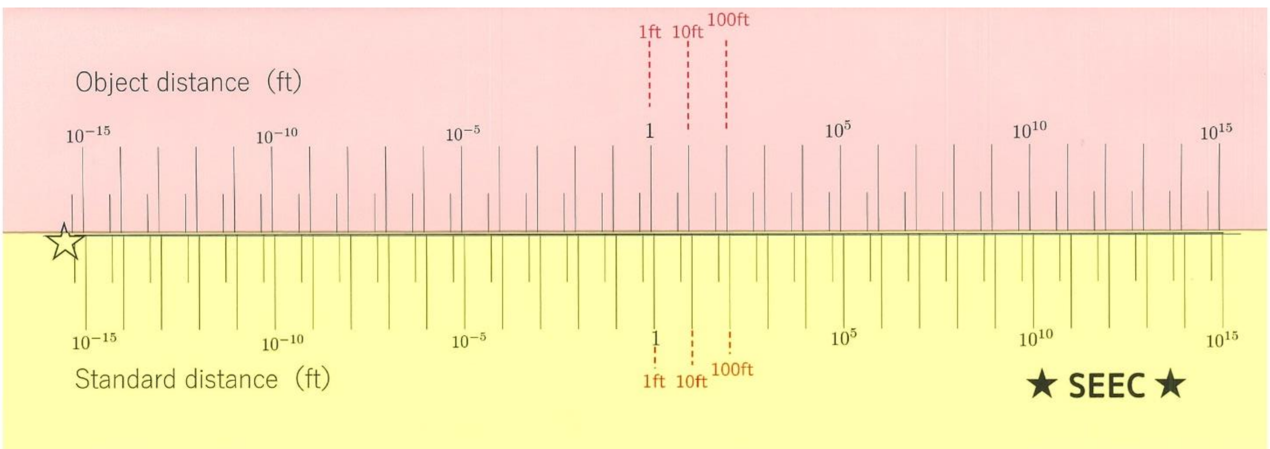
The logarithm ruler can used of in feet.

Question 1) If the earth's diameter were 1ft, how many long would the sun's diameter be?
The diameter of the earth equals 4.1×10^{7}ft
The diameter of the sun equals 4.5×10^{9} ft.

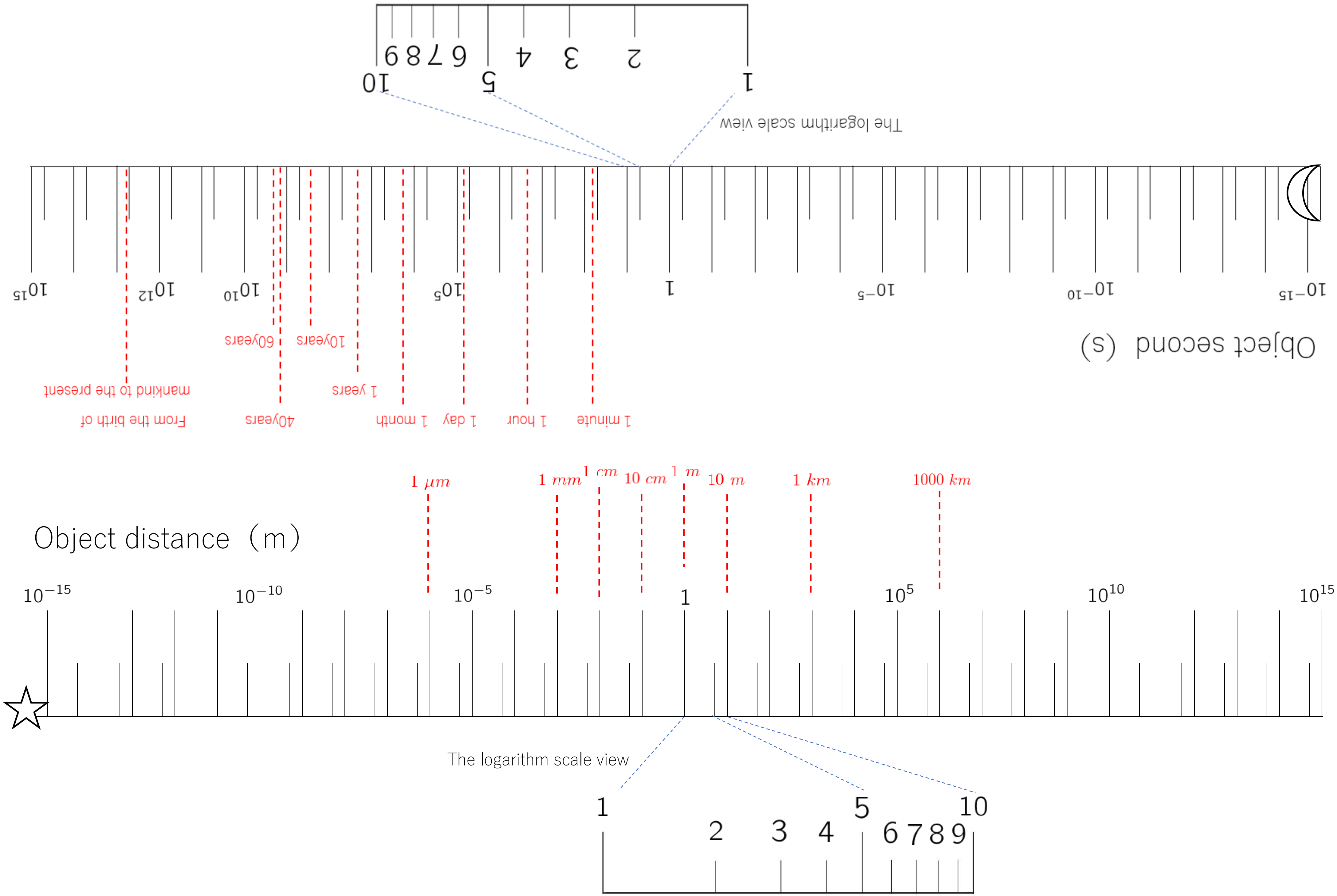
Question 2) If the distance from the earth to the moon were 1ft, how long would the distance from the earth to the sun be?
The distance from the earth to the moon equals 1.2×10^{9} ft
The distance from the earth to the sun equals 4.8×10^{11} ft

Question 3) If a virus were to become visible to us, how long would the diameter of a piece of chocolate be?
The diameter of a virus equals 3.28×10^{-7} ft
The diameter of a piece of chocolate equals 4.2×10^{-2} ft

Question 4) How long would you be alive if the time between the first human being to the present was one day?
Birth of human beings equals 6.2×10^{12} seconds

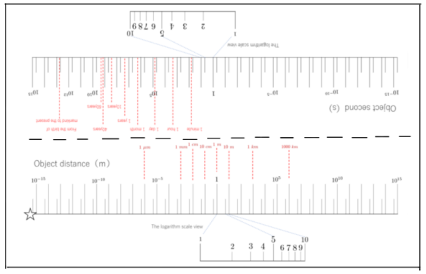


↑ The logarithm ruler in feet.

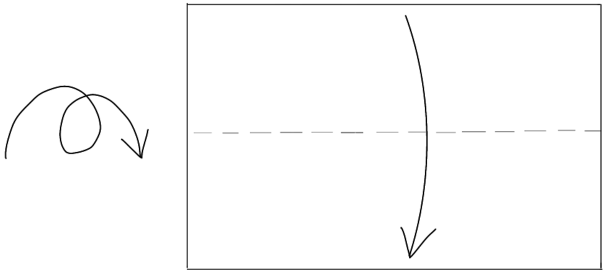


1
2
3
4
5
6 7 8 9
10
The logarithm scale view
10^{15}
10^{12}
10^{10}
10^{5}
1
10^{-5}
10^{-10}
10^{-15}
Object second (s)
60years
10years
From the birth of mankind to the present
1 years
40years
1 month
1 day
1 hour
1 minute
$1\ \mu m$
$1\ mm$
$1\ cm$
$10\ cm$
$1\ m$
$10\ m$
$1\ km$
$1000\ km$
Object distance (m)
10^{-15}
10^{-10}
10^{-5}
1
10^{5}
10^{10}
10^{15}
The logarithm scale view
1
2
3
4
5
6 7 8 9
10

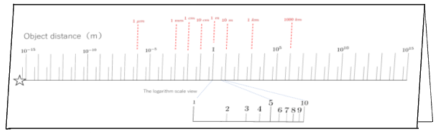
1

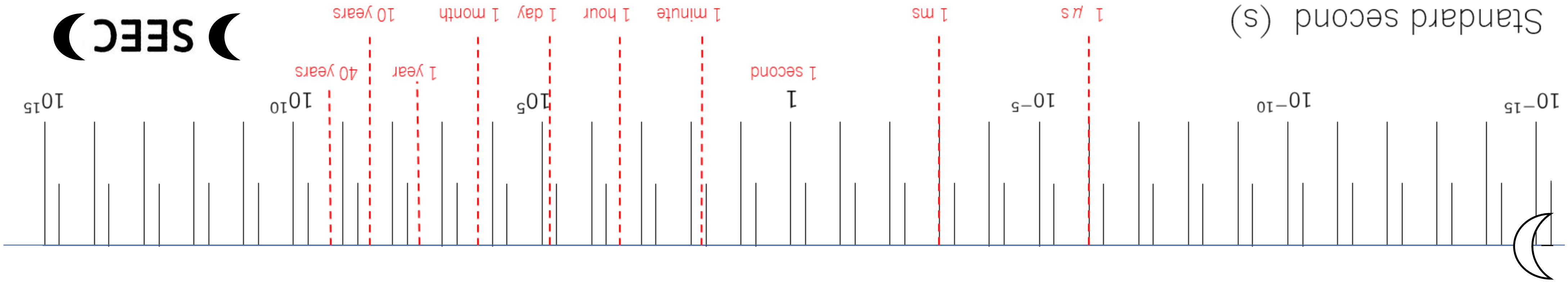


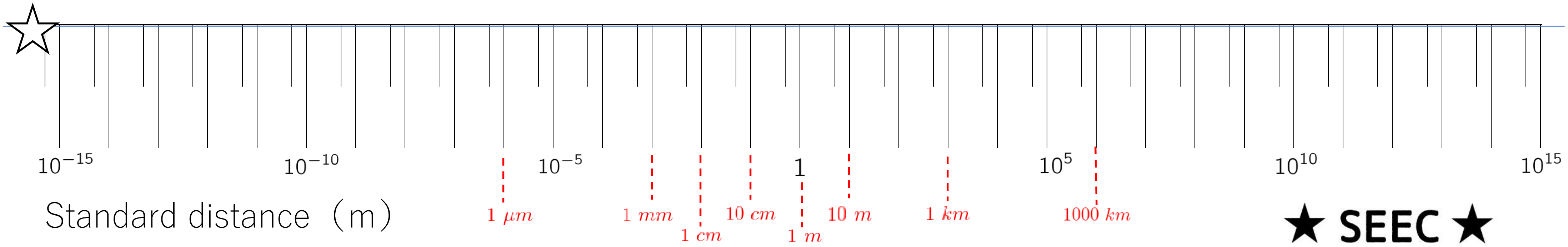
2



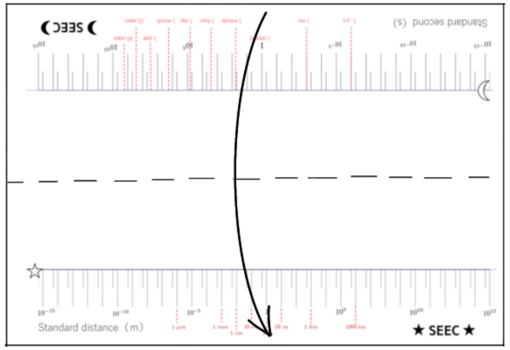
3



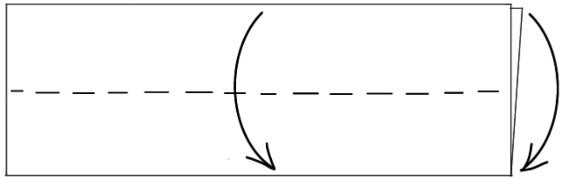

Standard second (s)
SEEC
1 μs
1 ms
1 minute
1 hour
1 day
1 month
10 years
1 second
1 year
40 years
10^{-15}
10^{-10}
10^{-5}
1
10^{5}
10^{10}
10^{15}


10^{-15}
10^{-10}
10^{-5}
1
10^{5}
10^{10}
10^{15}
1 μm
1 mm
1 cm
10 cm
1 m
10 m
1 km
1000 km
Standard distance (m)
★ SEEC ★

1



2



3